On the looseness of FO derandomization

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Abstract. This paper proves, for two examples of a randomized ROM PKE C, that derandomizing C degrades ROM OW-CPA security by a factor close to the number of hash queries. The first example can be explained by the size of the message space of C but the second cannot. This paper also gives a concrete example of a randomized non-ROM PKE C that appears to have the same properties regarding known attacks.

Keywords: public-key encryption, Fujisaki–Okamoto transformation, T transformation

1 Introduction

Fujisaki–Okamoto [28] proposed modularizing the task of designing a hopefully-IND-CCA2 PKE into two tasks:

- Design a hopefully-one-way PKE. This is a simpler task: one does not have to worry about distinguishers or about chosen-ciphertext attacks.
- Apply a generic transform, now called the "FO transform", to obtain a hopefully-IND-CCA2 PKE.

The usual argument for safety of the resulting PKE is as follows: (1) we believe, based on cryptanalysis, that the original PKE is in fact one-way ("OW-CPA"); (2) there is a theorem saying that if the original PKE is OW-CPA then the transformed PKE is ROM IND-CCA2; (3) we believe that there are no IND-CCA2 attacks more effective than ROM IND-CCA2 attacks.

However, even if the first and third steps in this argument are correct, a closer look shows that the FO theorem in the second step is not tight. The ROM IND-CCA2 advantage could be polynomially higher than the success probability of OW-CPA attacks against the original PKE.

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Subsequent work reviewed below has produced tight ROM theorems for some FO variants. A tight theorem typically follows one of three paths: (1) assume a *deterministic* OW-CPA PKE; (2) assume an *IND-CPA* PKE; (3) require a much less efficient iterated transform. But consider the following setting: a transform having similar efficiency to the original FO transform is applied to a *randomized* PKE, and one wants to deduce security from an OW-CPA assumption rather than making a stronger IND-CPA assumption. This setting appears frequently, and the theorems available for this setting are unsatisfactory.

1.1. Examples of randomized PKEs. Consider, e.g., the DH NIKE from [25], written here in additive notation as commonly used for elliptic curves: Alice publishes aG, where a is secret and G is public; Bob publishes bG, where b is secret; Alice and Bob now share a secret abG. Relabeling Bob's "public key" bG as "ciphertext" converts this NIKE into a KEM. Simple additive encryption of a message M converts this KEM into a PKE, the ElGamal PKE from [26], with ciphertext $(bG, M + abG)$. Decryption in this PKE recovers only M, not the randomness b that was used in encryption.

Similarly, in post-quantum cryptography, a typical construction of a hopefully-IND-CCA2 lattice-based PKE (see, e.g., [2]) starts from a randomized "noisy DH" PKE, with aG and bG replaced by $aG + e$ and $bG + d$. The construction then applies a reasonably efficient FO variant to convert this hopefully-OW-CPA PKE into a hopefully-IND-CCA2 PKE.

For original DH, common practice skips ElGamal's PKE, skips FO, takes the KEM described above, and modifies the KEM by simply applying a hash function as proposed by Shoup in [50], so the KEM ciphertext is bG and the KEM session key is $H(abG)$. But—even if one believes that this DH KEM is secure—it is not so easy to skip FO for typical lattice-based "noisy DH" PKEs. Alice computes $a(bG+d) = abG+ad$, while Bob computes $b(aG+e) = abG+be$, which is different. Alice applies an error-correction process to suppress the difference between ad and be, extracting the correct M in the end, but the same error-correction process allows easy chosen-ciphertext attacks that add small modifications to $bG + d$; see, e.g., [30] and [55]. It is thus unsurprising that FO is used pervasively in post-quantum cryptography.

Assuming IND-CPA for the underlying PKEs is more risky than assuming merely OW-CPA. The central issue is that distinguishing problems such as IND-CPA offer more attack avenues than search problems such as OW-CPA. As Goldreich wrote in [29]: "What concerns us about the DDH assumption is the fact that this assumption refers to a setting that is less simple than usual (e.g., DDH is less simple than DH), which makes this assumption harder to evaluate." See [8, Sections 6.2–6.3] for examples illustrating that mathematical algorithm designers focus primarily on search problems. The occasional studies of the extra risks of distinguishing problems have produced some easy breaks (e.g., DDH is broken when there are "cofactors", and decisional LWE for the polynomial $x^n - 1$ is broken by the factor $x - 1$ of $x^n - 1$) and some more subtle breaks (e.g., some elliptic curves have efficient pairings, breaking DDH), which is worrisome. So it

is important to ask, and the literature asks, whether an OW-CPA assumption suffices for a tight proof.

1.2. Is there a guarantee of ROM IND-CCA2 security for an efficient transform of a randomized OW-CPA PKE? The best ROM theorems available for this setting (see, e.g., [31, Section 3.3]) say that the IND-CCA2 advantage against the transformed system is bounded by about $2q$ times the OW-CPA success probability against the original system, when there are at most q calls to the oracle used for derandomization and at most q calls to other oracles. Note that Bitcoin currently carries out more than 2^{90} hashes per year, and one can easily imagine large-scale attackers devoting similar resources to cryptanalysis, so a factor q cannot simply be ignored.

One response is to adjust cryptosystem parameters to compensate. Bellare–Rogaway wrote in [6, Section 1.2]: "We reiterate the crucial point: if the reduction proving security is 'loose,' like the one above, the efficiency of the scheme is impacted, because we must move to a larger security parameter." However, this would damage efficiency and is not done by any of the current candidates in the NIST Post-Quantum Cryptography Standardization Project ("NISTPQC").

Another response is to avoid this setting: start from a deterministic PKE so as to be able to use tight proofs, as recommended in [10, Appendix F]. However, as noted in [38], only 2.5 of the 17 round-2 NISTPQC encryption candidates took this approach: Classic McEliece [9], NTRU [22], and the Streamlined NTRU Prime option within NTRU Prime [12].

Another response is to search for a tight theorem, eliminating the q factor in the current theorems. But perhaps the q factor cannot be eliminated. One scenario to consider is that the CCA transform maintains security without this being provable; many truths are unprovable. Another scenario to consider, an example of what Menezes [40] calls the "nightmare scenario" for loose proofs, is that CCA transforms make attacks q times easier.

1.3. Overview of FO improvements. Shoup [50, Section 4.2] (see also [52, Section 3]) proposed constructing a hopefully-IND-CCA2 PKE by constructing a hopefully-IND-CCA2 KEM and constructing a hopefully-secure DEM. KEMs are simpler than PKEs, and hopefully-secure DEMs are readily available from symmetric cryptography.

Dent [23] proposed constructing a hopefully-IND-CCA2 KEM by combining a hopefully-OW-CPA PKE with a CCA transform, analogous to the FO transform but in the simpler KEM context. Modern KEM constructions typically follow this structure, although the details of the CCA transform vary.

One of Dent's theorems [23, Theorem 8] obtains a ROM IND-CCA2 KEM *tightly* from any deterministic OW-CPA PKE, using what is now known as "plaintext confirmation". Persichetti [44, Section 5.3] introduced, for a particular deterministic PKE, a different strategy for tight ROM IND-CCA2 KEM proofs from OW-CPA, using what is now known as "implicit rejection". Hofheinz–Hövelmanns–Kiltz [31] generalized the implicit-rejection theorem to handle any deterministic OW-CPA PKE.

Hofheinz–Hövelmanns–Kiltz also observed that a wide range of FO variants for randomized PKEs factor into two simpler pieces, and presented state-of-the-art proofs factored analogously. The first piece is always the following transform T, called derandomization. The transform is given a PKE C and a public hash function H. The transform outputs a deterministic PKE $T(C, H)$, which is the same as C except that it uses $H(M)$ as the coins used to encrypt M. For example, if C is the ElGamal cryptosystem with a random choice of the DH scalar b, then $T(C, H)$ instead chooses $b = H(M)$.

Hofheinz–Hövelmanns–Kiltz proved tight ROM OW-CPA security of $T(C, H)$ assuming IND-CPA security of C, and loose ROM OW-CPA security of $T(C, H)$ assuming OW-CPA security of C. The loss factor is approximately q. An ℓ-fold iteration in [31, Section 3.4] obtains ROM IND-CPA security from OW-CPA security with loss factor only $q^{1/\ell}$ but makes ciphertexts ℓ times longer.

Further work on this topic includes allowing decryption failures in the PKE (see [31]), analyzing QROM IND-CCA2 security rather than just ROM IND-CCA2 security (see [31], [48], [34], [16], and [38]), additional factorizations of the transforms and proofs (see [48] and [14]), and various efforts to formally verify proofs (see, e.g., [53]).

All ROM IND-CCA2 theorems available today that start by merely assuming an OW-CPA PKE have loss factor at least $q^{1/\ell}$ with ℓ-fold ciphertext expansion; this is, *a fortiori*, also the case for QROM IND-CCA2 theorems. In particular, if the application is unwilling to incur a doubling of ciphertext size, the loss factor is at least q. Are better proofs possible? Or could it be that derandomization, the T transform, really does degrade OW-CPA security by a factor q?

1.4. Contributions of this paper. This paper reports the discovery of PKE examples where derandomization degrades OW-CPA security against standard attack strategies by a factor close to q. The PKE examples are reasonably simple, and the analyses are conceptually straightforward—no new cryptanalysis. The core novelty in this paper is *finding the examples*.

Three examples are presented. Example 2 is a ROM PKE with the feature of a *proof* that derandomization degrades OW-CPA security; this is a proof regarding *all* attacks, not just known attacks. Example 1 is a warmup for Example 2. For Example 3, there is no proof that the known attacks are optimal, but this example has the feature of being a concrete non-ROM PKE.

Section 2 proves, for both Example 1 and Example 2, that derandomization degrades OW-CPA security by a factor close to q. The derandomized OW-CPA success chance for Example 1 is $(q+1)/\#\text{Plaintexts}$; an attack with this success chance does not contradict typical notions of tightness. However, for Example 2, the OW-CPA success chance is far above $(q+1)/\#\text{Plaintexts}$.

Section 3 constructs Example 3, a non-ROM PKE for which derandomization appears, based on an analysis of known attacks, to degrade pre-quantum OW-CPA security by a factor close to q for every reasonable choice of hash function, where now q is the number of attack operations. As in Example 2, the success probabilities here are far above $(q+1)/\#\text{Plaintexts}$.

1.5. Consequences and open questions. It is tempting to imagine that one can selectively disregard limitations in proofs, treating a proof as evidence for something stronger than what the proof actually says. There is, for example, a proof that derandomization loosely preserves OW-CPA security; surely this is evidence that derandomization tightly preserves OW-CPA security. There is also a tight OW-CPA proof for derandomization under a stronger IND-CPA assumption; surely this is further evidence for the same hypothesis. But the hypothesis is incorrect: it is disproven by Example 2.

One way to avoid this disproof would be to retreat to the weaker hypothesis that derandomization tightly preserves OW-CPA security for *non-ROM* PKEs. But this hypothesis cannot have a relativizable proof,[3] given Example 2. The evidence provided *for* this hypothesis, namely extrapolation from weaker proofs, is relativizable, and relativizing the extrapolation produces a false statement, so the evidence is weak. Meanwhile this hypothesis implies a much better OW-CPA attack against Example 3, begging the question of what this attack is.

A time-tested approach to managing cryptographic risks requires all proof gaps to be filled with detailed cryptanalysis—or with better proofs. Could an OW-CPA assumption plus a small extra assumption produce a tight proof? Tight proofs are already known assuming IND-CPA, but IND-CPA assumptions are more risky than OW-CPA assumptions. Perhaps some sort of intermediate assumption can be identified that (1) suffices for a tight IND-CCA2 proof, (2) eliminates all of this paper's examples, and at the same time (3) follows from OW-CPA for some proposed cryptosystems, or at least is a simpler cryptanalytic target than IND-CPA. This paper's examples could help guide the search for such a proof, the same way that existing proofs helped guide the search for this paper's examples.

Regarding cryptanalysis, there are many randomized hopefully-OW-CPA PKE proposals in the literature. The obvious cryptanalytic challenge, in the absence of a tight proof for derandomization, is to understand the impact that derandomization has upon the security of each proposal. Often randomized PKEs are packaged with specific CCA transforms, but the analysis is important in any case. The CCA transforms are generic "plug and play" components, proposed for use with any PKE meeting specified rules; if this would degrade the security of a particular PKE then there needs to be a warning regarding this degradation.

[3] Retreating to statements about non-ROM PKEs also raises questions regarding whether these statements logically compose. Consider, e.g., the "noisy DH" lattice-based PKE mentioned above, presumably the target of cryptanalysis. This PKE chooses G randomly, but a real proposal typically chooses G as hash output from a short public string. This is not a problem for ROM analyses: one steps through (1) a ROM PKE that chooses G as random-oracle output, (2) a derandomized ROM PKE, (3) a ROM KEM including a CCA conversion, and, if desired, (4) a ROM PKE including this KEM and a DEM. If one of these steps is only for non-ROM cryptosystems, then logically one has to figure out whether this step can be at the beginning of the chain. If two steps are only for non-ROM cryptosystems then it is not at all clear what to do.

1.6. Choice of terminology. "OW-CPA" is renamed "OW-Passive" in [14], for the following reason: "The 'chosen-plaintext attacks' terminology is misleading: it suggests, incorrectly, that the attacker is permitted to choose plaintexts."

Some examples of IND-CPA being broken, without "OW-CPA" being broken, exploit the extra risks of distinguishers (IND) compared to search (OW); this is captured in the standard terminology. Other examples exploit the attacker's ability to choose plaintexts; this is not reflected in the standard terminology.

A counterargument says that security reviewers are overloaded, and that this is a serious problem for ongoing efforts to select post-quantum cryptosystems. Changing terminology adds to this load, at least in the short term, perhaps outweighing the advantages of more descriptive terminology. This paper says "OW-CPA".

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2 Derandomizing a generic information-leaking PKE

This section

- defines $\mathrm{GenericPKE}_{a,b,c,h}$, a randomized ROM PKE;
- specifies parameter choices for Example 1 and Example 2;
- shows that the q-query ROM OW-CPA insecurity of $\mathrm{GenericPKE}_{a,b,c,h}$ is exactly $1/2^b + q/2^{b+h}$, assuming $q \leq 2^b - 1$; and
- shows that the q-query ROM OW-CPA insecurity of the derandomized PKE $\mathrm{TGenericPKE}_{a,b,c,h}$ is at least $(q+1)/2^b$, again assuming $q \leq 2^b - 1$.

Derandomization thus degrades ROM OW-CPA insecurity of this ROM PKE by a factor at least $(q+1)2^h/(q+2^h)$. This factor is very close to $q+1$, under the reasonable assumption that 2^h is much larger than q. The message space Plaintexts for $\mathrm{GenericPKE}_{a,b,c,h}$ has size 2^{a+b}.

2.1. Overview of the PKE construction. If decryption time is irrelevant and unconstrained, the question "What is a simple example of a randomized ROM PKE?" is straightforwardly answered by the special case $a = 0$ of this section: $\mathrm{GenericPKE}_{0,b,c,h}$ feeds a b-bit plaintext and h bits of randomness to a random oracle, producing a c-bit ciphertext.

The further question "Why does OW-CPA not imply IND-CPA?" is standard. One of the standard answers is a PKE transformation that adds information to plaintexts, say an a-bit string ℓ, and adds the same information to ciphertexts. This has no effect on OW-CPA, but it breaks IND-CPA as soon as $a > 0$; if OW-CPA is achievable then this separates IND-CPA from OW-CPA. Applying this transformation to $\mathrm{GenericPKE}_{0,b,c,h}$ produces $\mathrm{GenericPKE}_{a,b,c,h}$.

Building PKEs from random oracles is not a new idea. See, e.g., the more complicated ROM PKEs used by Shoup in [51] to provide "strong evidence that the OAEP construction is not sound": those PKEs are secure, but applying OAEP to those PKEs destroys all security.[4] However, the consequences of such PKEs for derandomization do not appear to have been observed before.

2.2. Parameter requirements and examples. This paper restricts attention to b chosen so that 2^b is larger than the number of queries q allowed for the attacker. The comparisons of attack probabilities further assume that h is chosen so that 2^h is much larger than q; for concreteness, the reader can take $h = 2b$. Finally, the PKE construction requires $c \geq b + h$.

Example 1, this paper's first example, is $\text{GenericPKE}_{0,b,c,h}$, specializing this section to $a = 0$. For this example, $\#\text{Plaintexts} = 2^b$, so the original OW-CPA attack probability is only about $1/\#\text{Plaintexts}$, and the derandomized OW-CPA attack probability is only $(q + 1)/\#\text{Plaintexts}$; as noted in Section 1, this does not contradict typical notions of tightness.

Example 2 is $\text{GenericPKE}_{b,b,c,h}$, instead specializing this section to $a = b$. For this example, $\#\text{Plaintexts} = 2^{2b}$, so the original OW-CPA attack probability is approximately the square root of $1/\#\text{Plaintexts}$, and the derandomized OW-CPA attack probability is approximately q times larger than that.

2.3. The PKE. This subsection defines $\text{GenericPKE}_{a,b,c,h}$. This is a ROM PKE, using an oracle for a uniform random injective function F from $\{0,1\}^{b+h}$ to $\{0,1\}^c$. ROM success probabilities are by definition averaged over all choices of the oracle, along with all coin flips in algorithms.

Readers who prefer to work solely with uniform random functions, without injectivity constraints, can restrict attention to c much larger than $2(b + h)$, take F as a uniform random function, and observe that F is overwhelmingly likely to be injective. However, this would complicate the theorem statements to account for the tiny correctness error in the resulting PKE and the tiny chance of collisions spoiling the attack.

Definition 2.4. *Let a, b, c, h be nonnegative integers with $c \geq b + h$. Let F be a uniform random injective function from $\{0,1\}^{b+h}$ to $\{0,1\}^c$. Then* $\text{GenericPKE}_{a,b,c,h}(F)$ *is defined as*

$$(\text{PublicKeys}, \text{PrivateKeys}, \text{Plaintexts}, \text{Ciphertexts}, \text{KeyGen}, \text{Encrypt}, \text{Decrypt})$$

with the following components:

- $\text{PublicKeys} = \{0,1\}^0 = \{()\}$.
- $\text{PrivateKeys} = \{0,1\}^0 = \{()\}$.
- $\text{Plaintexts} = \{0,1\}^a \times \{0,1\}^b$.
- $\text{Ciphertexts} = \{0,1\}^a \times \{0,1\}^c$.
- KeyGen *is the following algorithm:*

[4] Shoup also gave a counting argument that the insecurity of OAEP on average over all oracles implies the existence of specific oracles relative to which OAEP is insecure.

  - *Input the empty string* ().
  - *Output* ((), ()).
- Encrypt *is the following algorithm:*
  - *Input* $((\ell, m), p) \in (\{0,1\}^a \times \{0,1\}^b) \times \{()\}$.
  - *Generate a uniform random* $r \in \{0,1\}^h$.
  - *Output* $(\ell, F(m,r)) \in \{0,1\}^a \times \{0,1\}^c$.
- Decrypt *is the following algorithm:*
  - *Input* $((\ell, z), s) \in (\{0,1\}^a \times \{0,1\}^c) \times \{()\}$.
  - *Search all* $(m,r) \in \{0,1\}^b \times \{0,1\}^h$ *in lexicographic order.*
  - *Output* (ℓ, m) *for the first* (m,r) *such that* $F(m,r) = z$.
  - *If no such* (m,r) *exists, output* $\perp$.

The decryption algorithm is very slow, but decryption is irrelevant to the OW-CPA security definition. This choice of decryption algorithm also removes the need for any randomness in private keys. There is also no need for any randomness in public keys, since there is enough randomness in F.

Theorem 2.5. *Under the assumptions of Definition 2.4,* $\text{GenericPKE}_{a,b,c,h}(F)$ *is a correct PKE.*

Proof. Syntactic requirements: PublicKeys, PrivateKeys, Plaintexts, Ciphertexts are nonempty finite sets; $\perp \notin$ Plaintexts; KeyGen is an algorithm mapping $\{()\}$ to PublicKeys $\times$ PrivateKeys; Encrypt is an algorithm mapping Plaintexts $\times$ PublicKeys to Ciphertexts; Decrypt is an algorithm mapping Ciphertexts $\times$ PrivateKeys to Plaintexts $\cup \{\perp\}$.

Correctness: Say KeyGen() outputs (p, s); $M \in$ Plaintexts; and Encrypt(M, p) outputs C. By definition of Plaintexts, $M = (\ell, m)$ for some $\ell \in \{0,1\}^a$ and $m \in \{0,1\}^b$. By definition of Encrypt, there is some $r \in \{0,1\}^h$ such that $C = (\ell, z)$ with $z = F(m,r)$. By assumption F is injective, so this (m,r) is the unique preimage of z under F. The search in Decrypt finds this preimage and outputs M as desired. □

2.6. Attacking the PKE. This subsection defines an OW-CPA attack against $\text{GenericPKE}_{a,b,c,h}$, and shows that the attack has success probability exactly $1/2^b + q/2^{b+h}$.

Definition 2.7. *Under the assumptions of Definition 2.4, let* q *be an element of* $\{0, 1, \ldots, 2^b - 1\}$*, and define* $\text{GenericAttack}_{a,b,c,h,q}(F)$ *as the following algorithm:*

- *Input* $(p, (\ell, z)) \in \{()\} \times (\{0,1\}^a \times \{0,1\}^c)$.
- *Generate a uniform random sequence of distinct elements* $m_0, m_1, \ldots, m_q$ *of* $\{0,1\}^b$.
- *Generate a uniform random sequence of elements* $r_1, \ldots, r_q$ *of* $\{0,1\}^h$.
- *For each* $i \in \{1, 2, \ldots, q\}$ *in increasing order: If* $F(m_i, r_i) = z$*, output* (ℓ, m_i) *and stop.*
- *Output* (ℓ, m_0).

Theorem 2.8. *Under the assumptions of Definition 2.7, the algorithm* $\mathrm{GenericAttack}_{a,b,c,h,q}(F)$ *uses at most* q *calls to the* F *oracle and has ROM OW-CPA success probability* $1/2^b + q/2^{b+h}$ *against* $\mathrm{GenericPKE}_{a,b,c,h}(F)$.

Proof. The algorithm calls the F oracle for $F(m_1, r_1)$; then, if $F(m_1, r_1) \neq z$, for $F(m_2, r_2)$; and so on through $F(m_q, r_q)$. This is at most q calls, and there are no other calls.

By definition the OW-CPA success probability of A against $\mathrm{GenericPKE}_{a,b,c,h}(F)$ is the chance that the following game outputs 1: compute $(p, s) \leftarrow \mathrm{KeyGen}()$; generate a uniform random $M \in \mathrm{Plaintexts}$; compute $C \leftarrow \mathrm{Encrypt}(M, p)$; output 1 if $A(p, C) = M$.

Write M as (ℓ, m). Then $C = (\ell, z)$ where $z = F(m, r)$ for some $r \in \{0,1\}^h$, by definition of Encrypt.

There is probability exactly $1/2^b$ that m_0 inside $A = \mathrm{GenericAttack}_{a,b,c,h,q}(F)$ matches m. If this occurs then by distinctness none of $m_1, \ldots, m_q$ match m, so, by injectivity of F, none of the outputs $F(m_i, r_i)$ match z, so A does not stop early, so A outputs $(\ell, m_0) = (\ell, m) = M$, and the OW-CPA game outputs 1.

There is also, for each $i \in \{1, 2, \ldots, q\}$, probability exactly $1/2^{b+h}$ that (m_i, r_i) inside A matches (m, r). If this occurs then by distinctness none of $m_1, \ldots, m_{i-1}$ match m, so, by injectivity of F, none of the outputs $F(m_1, r_1)$ through $F(m_{i-1}, r_{i-1})$ match z, so A does not stop before reaching this i; A then tries $F(m_i, r_i)$, which matches $F(m, r) = z$, so A outputs $(\ell, m_i) = (\ell, m) = M$, and again the OW-CPA game outputs 1.

Conversely, these events are the only way for the OW-CPA game to output 1: if $A(p, C) = M$ then either A outputs $(\ell, m_0) = M$ in the last step, in which case $m_0 = m$, or it outputs some $(\ell, m_i) = M$ in the previous step, in which case $m_i = m$.

Finally, these events are disjoint by distinctness of $m_0, \ldots, m_q$, so they occur with total probability $1/2^b + q/2^{b+h}$. □

2.9. Optimality of the attack. This subsection shows that, given its number of calls to the F oracle, the attack above reaches the maximum possible OW-CPA success probability against $\mathrm{GenericPKE}_{a,b,c,h}$. The fact that no attack can do better than probability $1/2^b + q/2^{b+h}$ against this PKE is what matters for seeing that derandomization damages security by a factor close to q; the fact that the specific attack above reaches probability $1/2^b + q/2^{b+h}$ shows that this OW-CPA analysis is complete.

The optimality proof relies on the fact that M is generated uniformly at random in the OW-CPA game, and that r is generated uniformly at random in Encrypt. These facts were not used in Theorem 2.8.

Theorem 2.10. *Under the assumptions of Definition 2.4, let* q *be an element of* $\{0, 1, \ldots, 2^b - 1\}$. *Every algorithm that uses at most* q *distinct calls to the* F *oracle has ROM OW-CPA success probability at most* $1/2^b + q/2^{b+h}$ *against* $\mathrm{GenericPKE}_{a,b,c,h}(F)$.

Proof. Let A be an algorithm using at most q distinct calls to the F oracle. Modify A to count the number of distinct oracle inputs and, just before stopping, add extra calls to F on uniform random inputs until the count reaches q; this will terminate since the domain of F has size $2^{b+h} \geq 2^b > q$. Now A makes exactly q distinct oracle calls.

In the OW-CPA attack game for A, there are $\prod_{0 \leq j < 2^{b+h}} (2^c - j)$ equally likely possibilities for the injective function F; then 2^{a+b} equally likely possibilities for (ℓ, m) from Plaintexts; and 2^h equally likely possibilities for r inside Encrypt(M, p), determining $C = (\ell, z)$ where $z = F(m, r)$.

A's initial view (p, C) reveals ℓ but provides no information about (m, r): for each choice of (m, r), there are exactly $\prod_{1 \leq j < 2^{b+h}} (2^c - j)$ choices of F satisfying $z = F(m, r)$. A's first oracle query (m_1, r_1), assuming $q \geq 1$, thus has $(m_1, r_1) = (m, r)$ with probability $1/2^{b+h}$, and $(m_1, r_1) \neq (m, r)$ with probability $1 - 1/2^{b+h}$.

Now condition on $(m_1, r_1) \neq (m, r)$. A's view after the oracle response z_1 provides no further information about (m, r): for each of the $2^{b+h} - 1$ choices of $(m, r) \neq (m_1, r_1)$, there are exactly $\prod_{2 \leq j < 2^{b+h}} (2^c - j)$ choices of F satisfying $z = F(m, r)$ and $z_1 = F(m_1, r_1)$. A's second distinct oracle query (m_2, r_2), assuming $q \geq 2$, thus has $(m_2, r_2) = (m, r)$ with conditional probability $1/(2^{b+h} - 1)$. The non-conditional probability that $(m_1, r_1) \neq (m, r)$ and $(m_2, r_2) \neq (m, r)$ is $1 - 2/2^{b+h}$.

Continue in the same way through all q distinct oracle queries. By induction, the total probability that $(m_1, r_1) \neq (m, r)$ and so on through $(m_i, r_i) \neq (m, r)$ is $1 - i/2^{b+h}$. A's view after oracle responses $z_1, \ldots, z_i$ provides no further information about (m, r): there are $2^{b+h} - i$ choices of (m, r) different from $(m_1, r_1), \ldots, (m_i, r_i)$, each produced by the same number of choices of F. A's next distinct oracle query (m_{i+1}, r_{i+1}), assuming $q \geq i + 1$, thus has $(m_{i+1}, r_{i+1}) = (m, r)$ with conditional probability $1/(2^{b+h} - i)$ if (m_{i+1}, r_{i+1}), i.e., non-conditional probability $1/2^{b+h}$, completing the induction for $i + 1$.

In particular, the total probability that $(m_1, r_1) \neq (m, r)$ and so on through $(m_q, r_q) \neq (m, r)$ is $1 - q/2^{b+h}$, and if this occurs then A's view after all q oracle responses provides no further information about (m, r). There are $2^{b+h} - q$ choices of (m, r) remaining at this point, and *at most* 2^h of them have (ℓ, m) matching the output from A, so A succeeds with conditional probability at most $2^h/(2^{b+h} - q)$; i.e., the non-conditional probability that A succeeds with $(m_1, r_1) \neq (m, r)$ and so on through $(m_q, r_q) \neq (m, r)$ is at most $1/2^b$. Meanwhile the probability that A succeeds with (m, r) matching one of $(m_1, r_1), \ldots, (m_q, r_q)$ is at most $q/2^{b+h}$. The total probability that A succeeds is at most $1/2^b + q/2^{b+h}$. □

2.11. The derandomized PKE. To keep this paper self-contained, this subsection defines TGenericPKE$_{a,b,c,h}$. The transformation from GenericPKE$_{a,b,c,h}$ to TGenericPKE$_{a,b,c,h}$ is an example of the standard T derandomization process from the literature.

Definition 2.12. *Under the assumptions of Definition 2.4, let H be a uniform random function from $\{0,1\}^{a+b}$ to $\{0,1\}^h$, and assume that F and H are inde-*

pendent. Then $\mathrm{TGenericPKE}_{a,b,c,h}(F,H)$ *is defined as*

$$(\mathrm{PublicKeys}, \mathrm{PrivateKeys}, \mathrm{Plaintexts}, \mathrm{Ciphertexts}, \mathrm{KeyGen}, \mathrm{TEncrypt}, \mathrm{Decrypt})$$

where TEncrypt *is the following algorithm:*

- *Input* $((\ell, m), p) \in (\{0,1\}^a \times \{0,1\}^b) \times \{()\}$.
- *Compute* $r = H(\ell, m) \in \{0,1\}^h$.
- *Output* $(\ell, F(m,r)) \in \{0,1\}^a \times \{0,1\}^c$.

$\mathrm{TGenericPKE}_{a,b,c,h}$ is the same as $\mathrm{GenericPKE}_{a,b,c,h}$ except for replacing Encrypt with TEncrypt. The only difference between Encrypt and TEncrypt is that Encrypt generates r uniformly at random while TEncrypt generates r as $H(M)$, where $M = (\ell, m)$ is the plaintext being encrypted.

2.13. Attacking the derandomized PKE. This subsection defines an OW-CPA attack against $\mathrm{TGenericPKE}_{a,b,c,h}$, and shows that the attack has success probability $(q+1)/2^b$, where q is the number of calls to the H oracle and also the number of calls to the F oracle. This completes the proof that derandomizing $\mathrm{GenericPKE}_{a,b,c,h}$ damages security by a factor close to q.

To also complete the analysis of OW-CPA security of $\mathrm{TGenericPKE}_{a,b,c,h}$, one could ask for a proof that the following attack is optimal, but it is easier to observe that near-optimality follows from composing existing T theorems with Theorem 2.10.

Definition 2.14. *Under the assumptions of Definition 2.12, let* q *be an element of* $\{0, 1, \ldots, 2^b - 1\}$*, and define* $\mathrm{TGenericAttack}_{a,b,c,h,q}(F,H)$ *as the following algorithm:*

- *Input* $(p, (\ell, z)) \in \{()\} \times (\{0,1\}^a \times \{0,1\}^c)$.
- *Generate a uniform random sequence of distinct elements* $m_0, m_1, \ldots, m_q$ *of* $\{0,1\}^b$.
- *For each* $i \in \{1, 2, \ldots, q\}$ *in increasing order: If* $F(m_i, H(\ell, m_i)) = z$*, output* (ℓ, m_i) *and stop.*
- *Output* (ℓ, m_0).

In $\mathrm{TGenericAttack}_{a,b,c,h,q}$, each of the guesses $m_1, \ldots, m_q$ is correct with chance $1/2^b$—which, again, is much larger than $1/\#\mathrm{Plaintexts} = 1/2^{a+b}$ when a is large—and, critically, derandomization allows each of these guesses to be checked efficiently. For comparison, in $\mathrm{GenericAttack}_{a,b,c,h,q}$, each of the guesses $m_1, \ldots, m_q$ is correct with chance $1/2^b$, but checking a guess for m involves also guessing r, reducing the success chance of each guess to $1/2^{b+h}$.

Theorem 2.15. *Under the assumptions of Definition 2.14, the algorithm* $\mathrm{TGenericAttack}_{a,b,c,h,q}(F)$ *uses at most* q *calls to the* F *oracle, uses at most* q *calls to the* H *oracle, and has ROM OW-CPA success probability* $(q+1)/2^b$ *against* $\mathrm{TGenericPKE}_{a,b,c,h}(F)$.

Proof. As in Theorem 2.8, except that r_i is replaced by $H(\ell, m_i)$ and the success probabilities are adjusted accordingly. Full details are spelled out here to aid in verification.

The algorithm calls the H oracle and then the F oracle for $F(m_1, H(\ell, m_1))$; then, if the output was not z, for $F(m_2, H(\ell, m_2))$; and so on. This is at most q calls to H and at most q calls to F. There are no other oracle calls.

By definition the OW-CPA success probability of A against $\mathrm{TGenericPKE}_{a,b,c,h}(F)$ is the chance that the following game outputs 1: compute $(p, s) \leftarrow \mathrm{KeyGen}()$; generate a uniform random $M \in \mathrm{Plaintexts}$; compute $C \leftarrow \mathrm{TEncrypt}(M, p)$; output 1 if $A(p, C) = M$.

Write M as (ℓ, m). Then $C = (\ell, z)$ where $z = F(m, H(\ell, m))$, by definition of TEncrypt.

There is probability exactly $1/2^b$ that m_0 inside $A = \mathrm{TGenericAttack}_{a,b,c,h,q}(F)$ matches m. If this occurs then by distinctness none of $m_1, \ldots, m_q$ match m, so, by injectivity of F, none of the outputs $F(m_i, H(\ell, m_i))$ match z, so A does not stop early, so A outputs $(\ell, m_0) = (\ell, m) = M$, and the OW-CPA game outputs 1.

There is also, for each $i \in \{1, 2, \ldots, q\}$, probability exactly $1/2^b$ that m_i inside A matches m. If this occurs then by distinctness none of $m_1, \ldots, m_{i-1}$ match m, so, by injectivity of F, none of the outputs $F(m_1, H(\ell, m_1))$ through $F(m_{i-1}, H(\ell, m_{i-1}))$ match z, so A does not stop before reaching this i; A then tries $F(m_i, H(\ell, m_i))$, which matches $F(m, H(\ell, m)) = z$, so A outputs $(\ell, m_i) = (\ell, m) = M$, and again the OW-CPA game outputs 1.

Conversely, these events are the only way for the OW-CPA game to output 1: if $A(p, C) = M$ then either A outputs $(\ell, m_0) = M$ in the last step, in which case $m_0 = m$, or it outputs some $(\ell, m_i) = M$ in the previous step, in which case $m_i = m$.

Finally, these events are disjoint by distinctness of $m_0, \ldots, m_q$, so they occur with total probability $(q + 1)/2^b$. □

3 Derandomizing a concrete PKE

Every ROM proof raises the question of whether the conclusion is an artifact of the ROM, i.e., whether extrapolating to concrete non-ROM proposals produces incorrect conclusions. Proofs generally do not address this question, so one falls back on cryptanalysis, searching for attacks against concrete proposals.

This section gives an example of a concrete PKE for which derandomization damages the pre-quantum OW-CPA security of the PKE against known attacks. The damage is quantitatively similar to what happens in the second example in Section 2: derandomization makes known attacks easier by a factor growing linearly with the number of operations available to the attacker.

This is *not* a theorem regarding all attacks; it is conceivable that better attacks could change the status of this PKE. A close inspection also shows that, as in other areas of cryptanalysis, the attack analyses rely on unproven conjectures. But any argument that derandomization is not risky needs to explain how the

argument is compatible not just with the proven ROM examples from Section 2 but also with the concrete example in this section.

This example is selected to rely entirely on well-known design techniques and well-known cryptanalytic techniques, reducing the chance of errors in the attack analysis. One could instead systematically survey previously published examples of PKEs and explore whether derandomization degrades the security of those PKEs; in general, this would be asking for new cryptanalysis, although there might be cases where attacks turn out to be as easy to write down as they are in this paper.

3.1. Is ElGamal an example? Consider again the ElGamal PKE, with public key aG and ciphertext $(bG, M + abG)$, with a standard group as the plaintext space. Assume for simplicity that $\langle G \rangle$ is the whole group, not a proper subgroup.

As in Section 2, the attacker can enumerate guesses for (M, b), and, if this fails, output a final guess for M. Checking q guesses for (M, b) has success chance $q/\#\langle G \rangle^2$ and takes q simple operations. The final guess for M has success chance $1/\#\langle G \rangle$, which is dominant under the reasonable assumption that q is small compared to $\#\langle G \rangle$. Derandomization, choosing b as a hash of M, increases the success chance to $(q+1)/\#\langle G \rangle$.

One can object that this is not a tightness problem: the attack has success chance only $(q+1)/\#\text{Plaintexts}$. However, modifying the PKE as in Section 2 to include additional information in plaintexts, leaked through ciphertexts, makes #Plaintexts much larger than $\#\langle G \rangle$, removing this objection. What matters is the success-probability ratio between attacks against the derandomized system and attacks against the original system.

A more serious objection is that there are much better attacks that instead spend q operations trying to compute the discrete logarithm a of aG. Even for our (conjecturally) strongest groups, generic attacks have success probability on the scale of $q^2/\#\langle G \rangle$, which is much larger than the probabilities $1/\#\langle G \rangle$ and $(q+1)/\#\langle G \rangle$ mentioned above. One is then faced with the question of whether derandomization allows q-operation attacks with higher success probability. This question does not appear to have been addressed in the cryptanalytic literature, so this paper moves on to another example.

3.2. Minimizing randomness in ElGamal plaintexts. A standard design technique in cryptography is to

- identify options for a specific component of a cryptographic system,
- restrict attention to options that reach a specified security level against known attacks according to a specified security metric, and
- choose the smallest option in a specified size metric.

The smallest option is typically described as being "efficient", while larger options are described as "wasting resources", being "overkill", etc. Consider, e.g., [3, Section 5] proposing usage of reduced-round ciphers "for a future where less energy is wasted on computing superfluous rounds".

Often this minimization of a cryptographic component is combined with an argument that larger options do not increase overall system security[5] beyond the specified constraint, given attacks against other components of the system. The larger options are then criticized as, e.g., being "unbalanced". NIST's official key-size recommendations for many years stated [4, Section 5.6.3] that combining "non-comparable strength" algorithms was "generally discouraged". For users of 256-bit ECC, this discouraged use of AES-256 and encouraged use of AES-128 instead, based on a security metric where AES-128 has "comparable strength" to 256-bit ECC while AES-256 has much higher strength.

This design approach often reduces security, for example because the specified security metric was too narrow. See, e.g., [7] showing that NIST's comparison between AES-128 and 256-bit ECC relies on considering only high-probability single-target attacks and fails when one considers a broader class of attacks. This section exploits a similar gap between different notions of security, after applying the following minimization to one component of the ElGamal PKE.

Consider the typical use of a PKE to communicate a random k-bit session key to achieve "k bits of security": for example, an AES-128 key for $k = 128$. ElGamal's plaintext M is not simply a k-bit key: it is a full-size group element, with many more than k bits of entropy—typically at least $2k$ bits, and sometimes even more to protect against known or suspected improvements in discrete-log attacks.

It is straightforward—see Example 3 below—to modify the ElGamal PKE for an "optimally efficient" plaintext space, the set $\{0,1\}^k$ of k-bit strings, exactly the set of session keys that the user wants to communicate. For comparison, the original message-space size "wastes precious randomness resources"; it is "overkill"; it is "unbalanced", since security of the whole PKE is certainly far below the group size.

This ElGamal modification is a simple example of cryptographic-component minimization. The component at issue is Plaintexts, the set of plaintexts. The specified security requirement for this component is that a guess for a secret (uniform random) plaintext succeeds with chance at most $1/2^k$. The size metric for this component is #Plaintexts. Certainly 2^k is smaller than $\#\langle G\rangle$. This ElGamal minimization is not new (see, e.g., [20, Section 5.1], using ElGamal to encrypt an encoding of a short session key); what is new here is the connection to derandomization.

3.3. Example 3: encoded-plaintext elliptic-curve ElGamal. Consider, in general, replacing ElGamal's $M + abG$ with $E(M) + abG$, where E is a public injection from Plaintexts to $\langle G\rangle$, easy to compute and easy to invert.

The special case Plaintexts $= \langle G\rangle$, with E as the identity map, is the original ElGamal system. As explained above, the generalization allows more "efficient" (meaning smaller) choices of #Plaintexts: specifically, #Plaintexts $= 2^k$ while $\#\langle G\rangle$ remains much larger than 2^k.

[5] Meanwhile the overall system *cost* rarely appears in the efficiency analysis.

Take, in particular, Plaintexts $= \{0,1\}^k$, and define E as the composition of the following three steps:

- Zero-pad the k-bit input to $2h \geq k$ bits.
- Map a $2h$-bit string (x_0, x_1) to a $2h$-bit string (x_4, x_5) defined by $x_2 = x_0 \oplus H(x_1)$, $x_3 = x_1 \oplus H(x_2)$, $x_4 = x_2 \oplus H(x_3)$, and $x_5 = x_3 \oplus H(x_4)$ where H is a standard h-bit hash function.
- Use Elligator [13] to map a $2h$-bit string invertibly to a point on a $(2h+1)$-bit elliptic curve.

Finally, Example 3 is this cryptosystem with a extra bits added into plaintexts and copied into ciphertexts, so that #Plaintexts $= 2^{a+k}$.

The middle step in E is an example of what Rivest [46] dubbed an "all-or-nothing transform". This particular transform is from earlier work by Johnson–Matyas–Peyravian [35], adding more rounds to the transform used by Bellare–Rogaway [5] inside OAEP. If H were secret then this transform would instead be called a 4-round Feistel cipher.

When the elliptic curve is chosen according to standard criteria, the best discrete-log attack known has success probability on the scale of $q^2/\#\langle G\rangle \approx q^2/2^{2h+1}$ after q simple operations. If the discrete-log computation fails, a final guess for M succeeds with probability $1/2^k$. If parameters are chosen so that $2^k > q$ and, e.g., $2h > 3k + 10$ then the overall success probability is only slightly above $1/2^k$.

For the derandomized version of the same PKE, the attacker does much better by trying q guesses for M. The success probability of this attack is $q/2^k$ (plus $1/2^k$ if a random final output is included); i.e., approximately q times larger than the success probability of the attack against the randomized PKE. Instead of spending effort on a low-probability discrete-log computation, the attacker spends the same effort exploiting derandomization to check higher-probability guesses for M.

This is not the end of the analysis, since one still has to check whether there is a better attack against the randomized PKE. Standard curve criteria allow small cofactors, such as 4 or 8, and Elligator requires a cofactor. It is well known that the ElGamal PKE is not IND-CPA in the presence of these cofactors: the attacker learns the bottom 2 or 3 bits of a and b, partitioning the set of curve points $E(M)$ into 4 or 8 immediately recognizable classes. However, this merely allows the attacker to exclude approximately 3/4 or 7/8 of the possibilities for M (assuming E is well distributed across classes). This lets the attacker reach success probability approximately $4/2^k$ or $8/2^k$ by checking (on average) 4 or 8 possibilities for the final guess M, but this is not a powerful enough distinguisher to allow productive use of q guesses for M.

Could there be a stronger DDH attack? If the curve happens to allow a fast pairing then one can much more reliably check a guess for M—in other words, a guess for abG—by checking whether the pairing output for (G, abG) matches the pairing output for (aG, bG). However, standard curve criteria eliminate all curves where efficient pairings are known.

Section 1 noted the relatively low cryptanalytic attention to distinguishers as a reason that making IND-CPA assumptions is riskier than making OW-CPA assumptions. For the same reason, it is risky to assume that there is no DDH attack strong enough to invalidate this example. However, derandomization damages security of this example against *known* attacks.

3.4. Variants. One can replace the elliptic curve above with a multiplicative group $(\mathbf{Z}/p)^*$, where p is prime, and replace Elligator with simply viewing a $2h$-bit string as an integer between 1 and 2^{2h}. Known discrete-log attacks take time subexponential in $\log p$, but it is straightforward to take $\log p$ large enough that these attacks have success chance below $1/2^k$, assuming standard conjectures.

If $(p-1)/2$ is also prime then the cofactor is just 2. If also $p > 2^{2h+1}$ then one can square each integer modulo p and work in the subgroup of squares, with cofactor 1; this encoding function in the ElGamal context appears in, e.g., [27, Section 2.2].

Finally, one can construct examples that build a group element M in two parts, where one part ℓ is leaked through a larger cofactor while the other part m is limited to 2^k possibilities. This avoids the need to insert an extra string ℓ into plaintexts and ciphertexts. It is easy to construct multiplicative groups with a specified cofactor, by searching for primes p in an arithmetic progression. For elliptic-curve groups, the techniques of Bröker–Stevenhagen [19] allow efficient construction of a group of order N, given any N that factors into powers of a small number of known primes.

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