How to
manipulate curve standards:
a white paper for the black hat

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Tanja Lange
Ruben Niederhagen
Christine van Vredendaal

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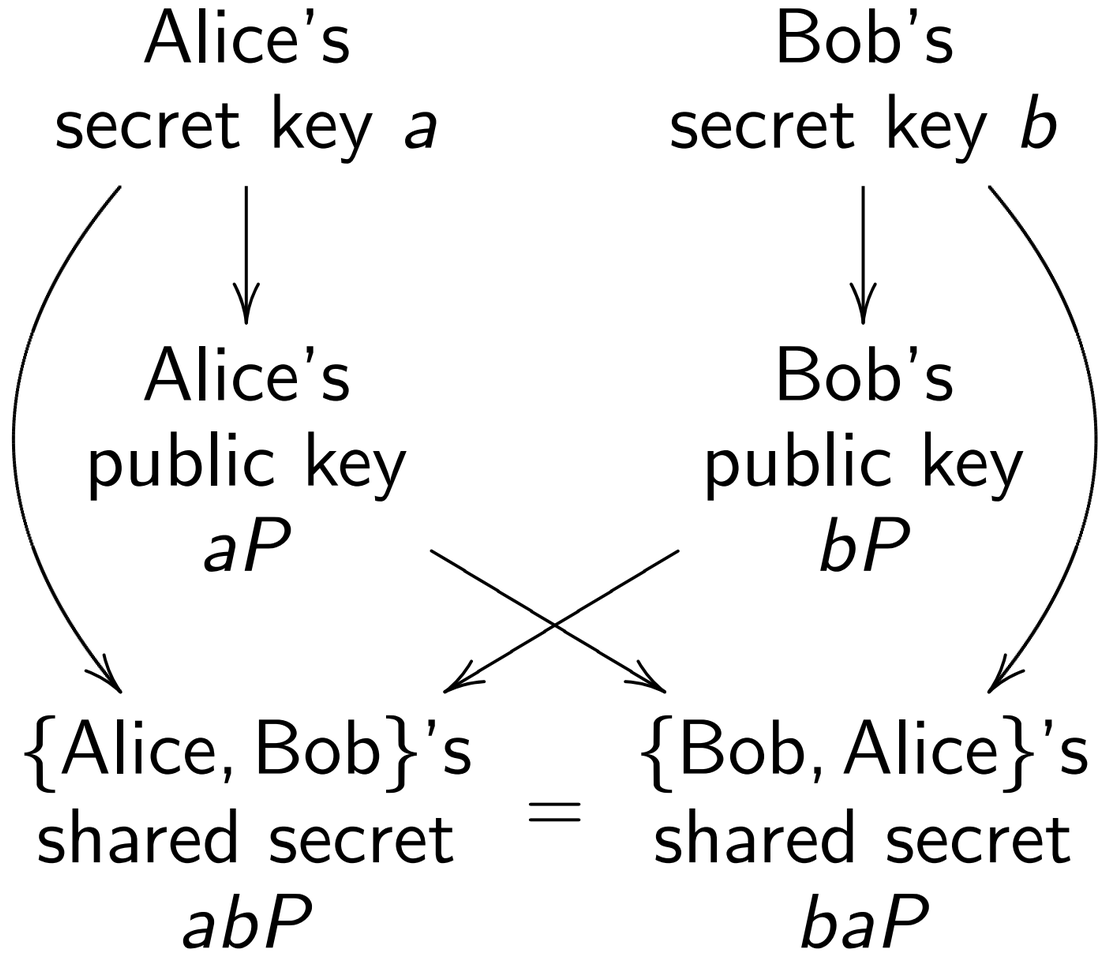
Textbook key exchange
using standard point P
on a standard elliptic curve E:



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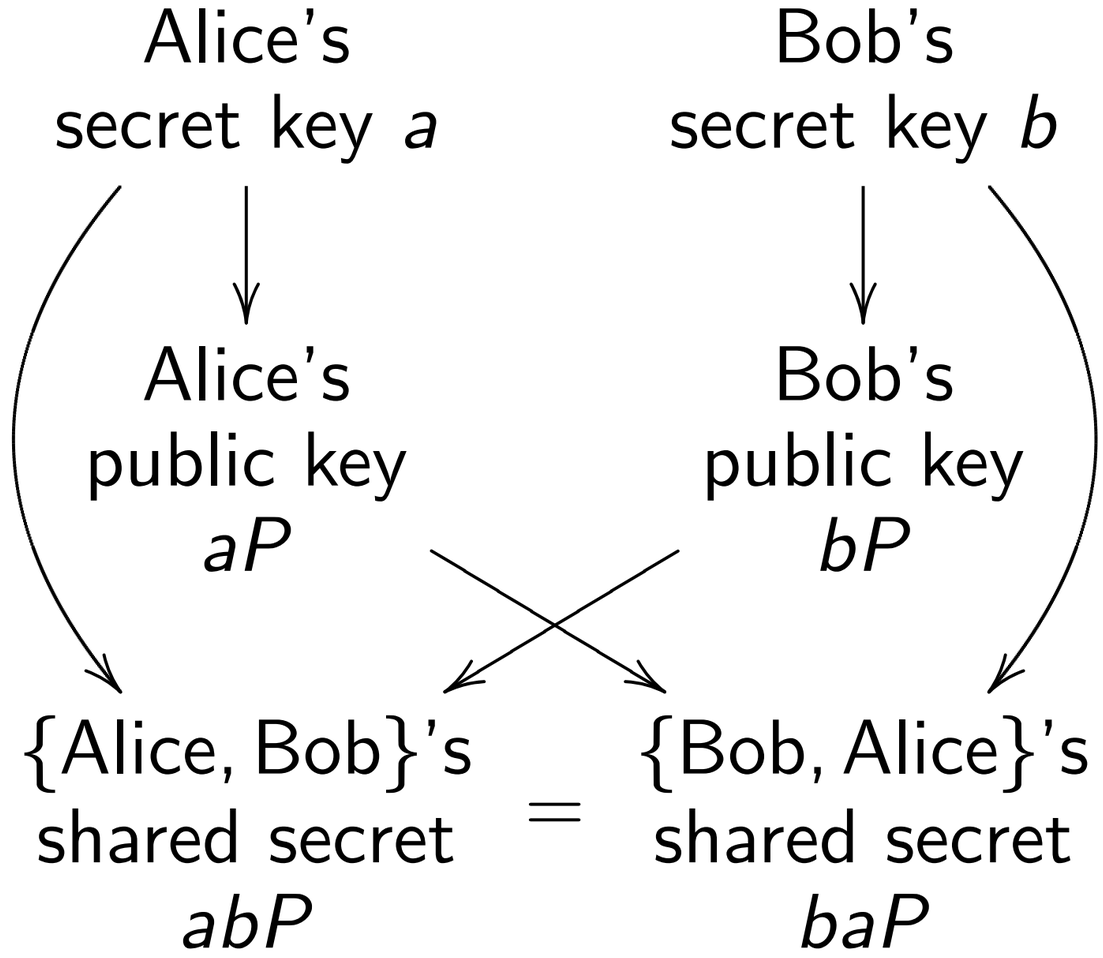

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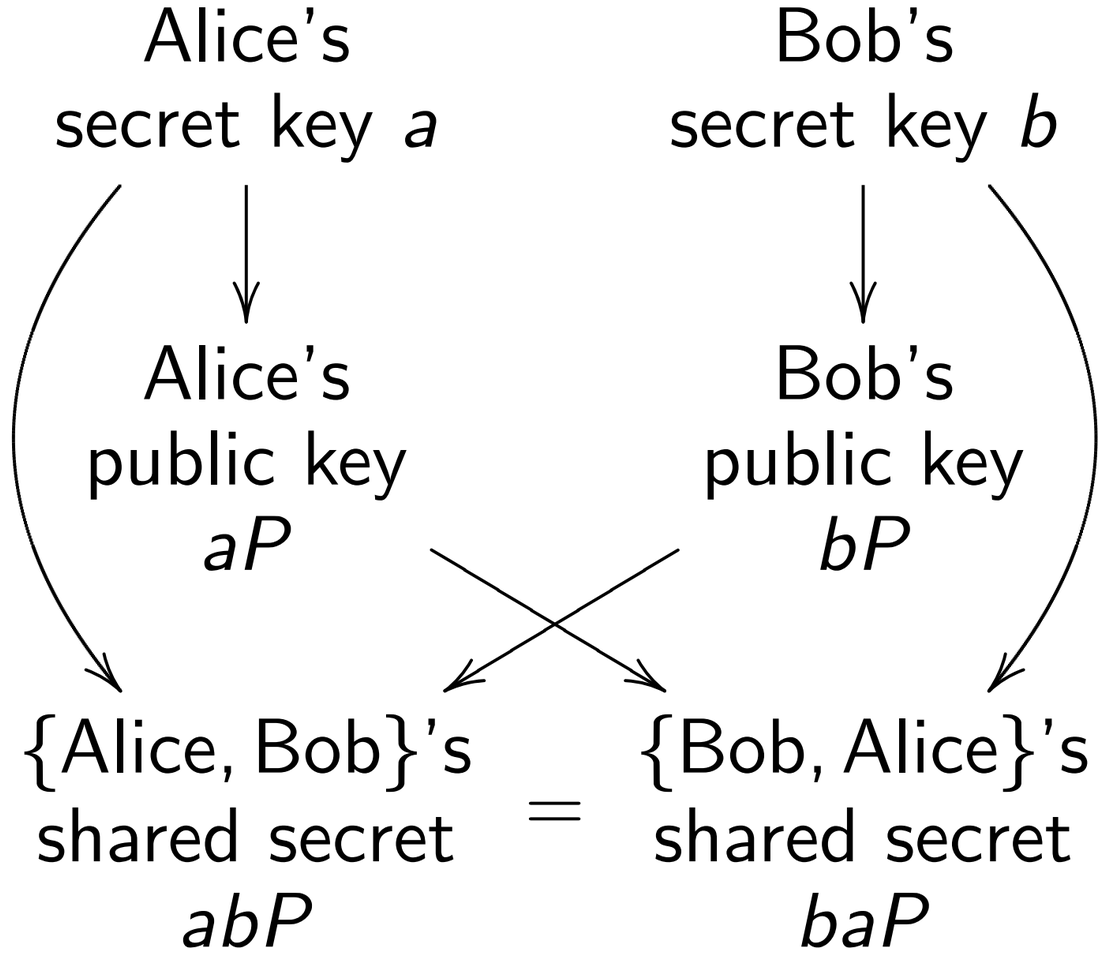

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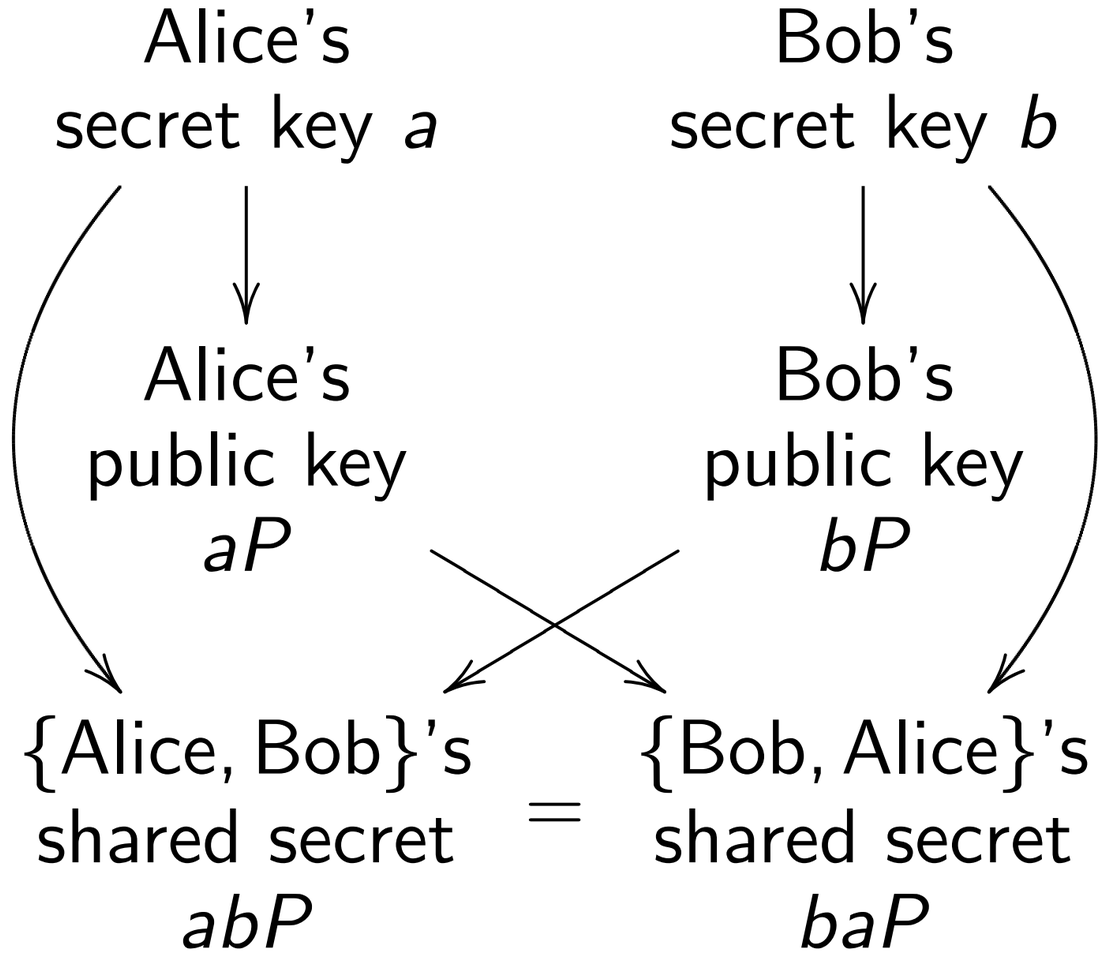

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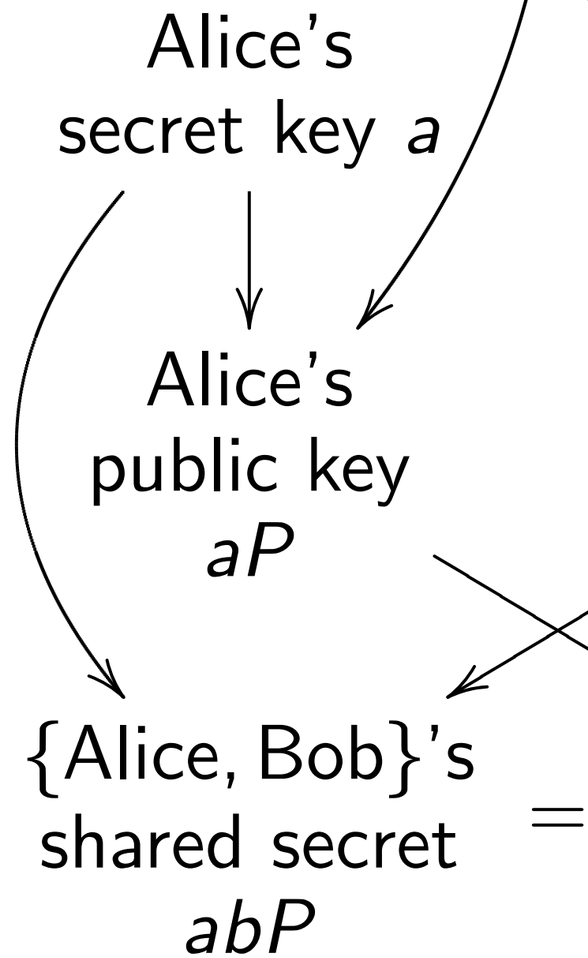

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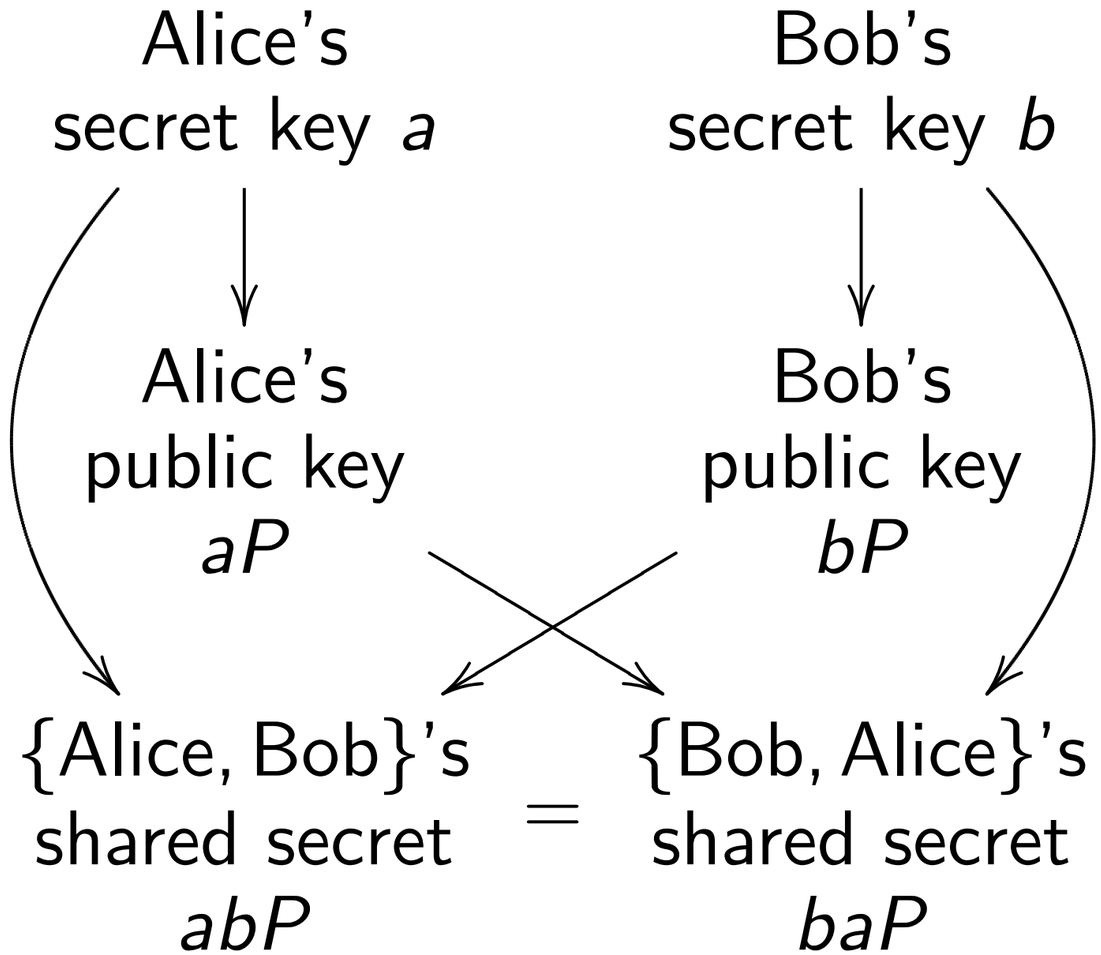

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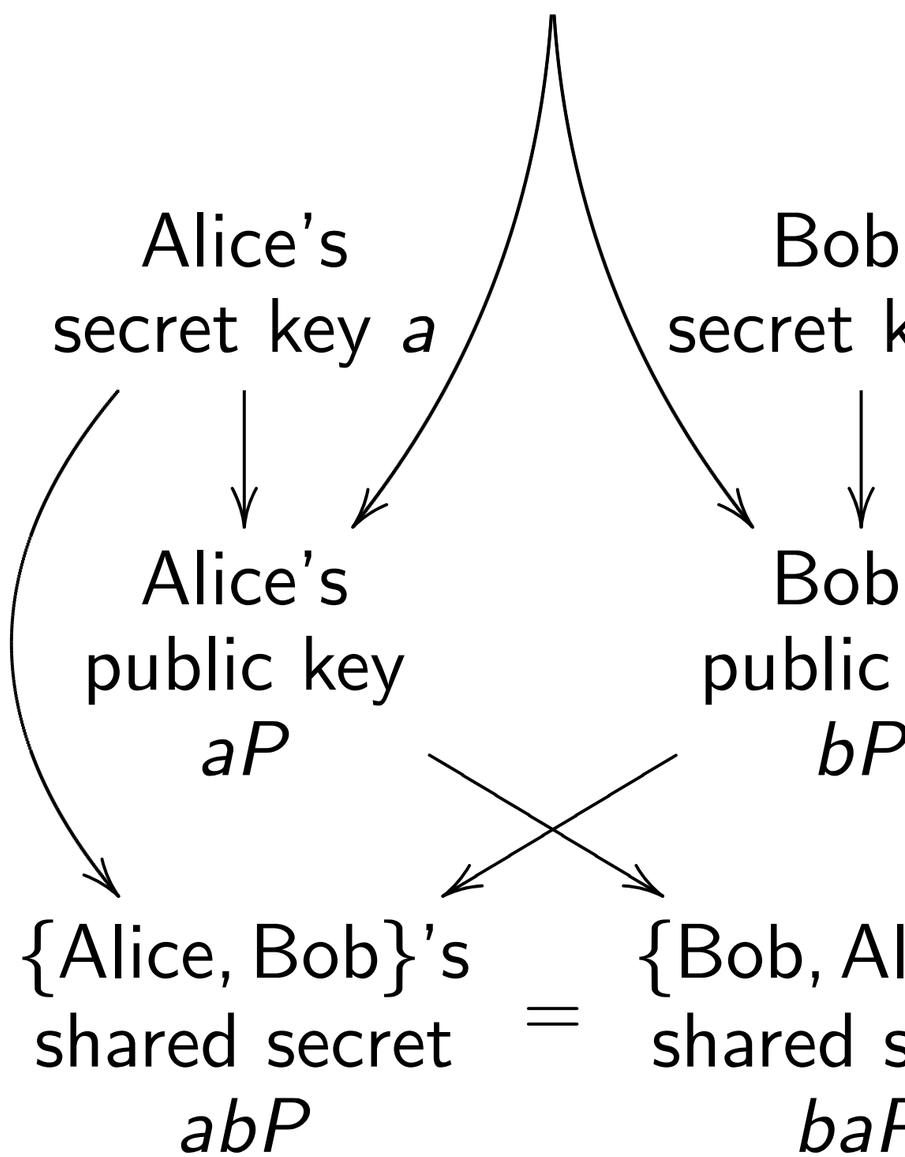

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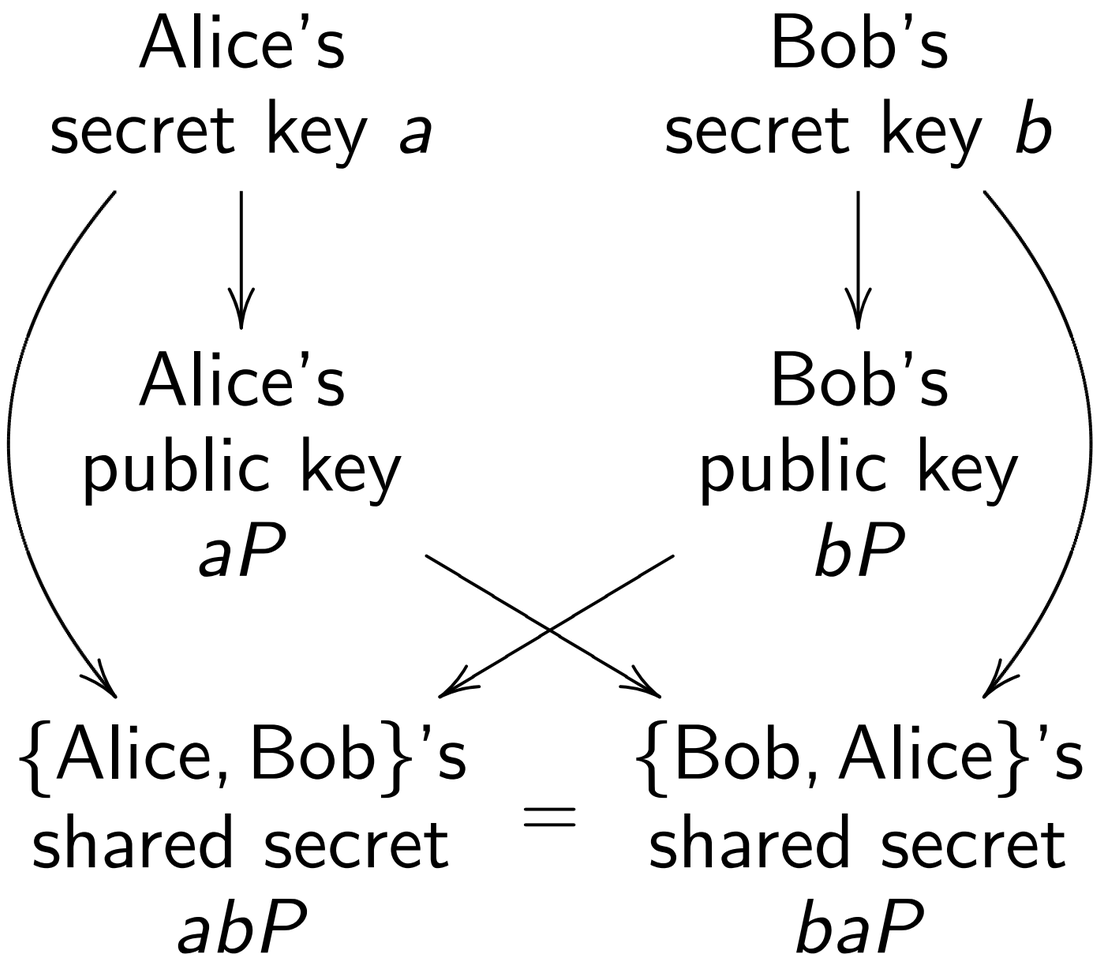

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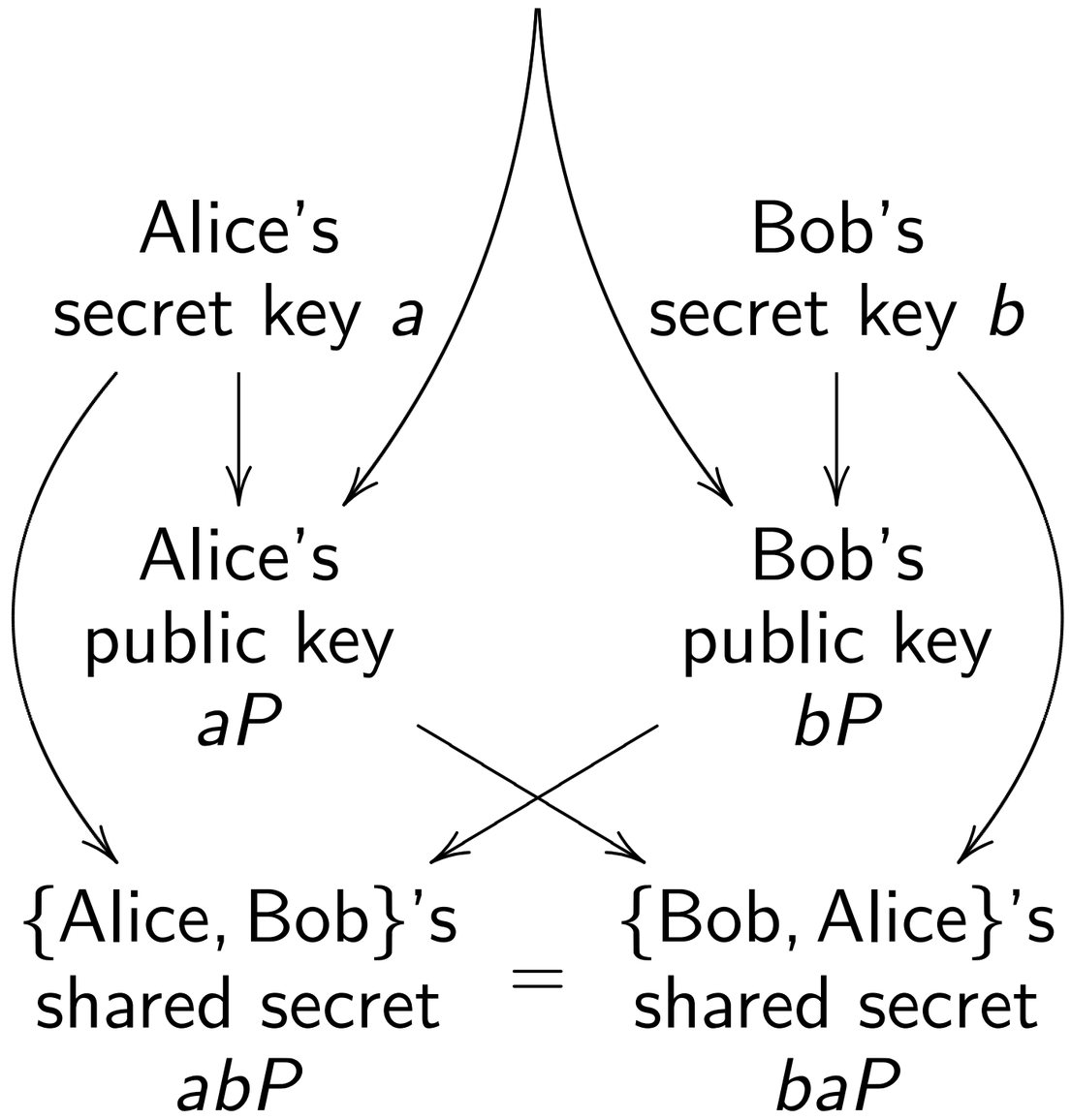

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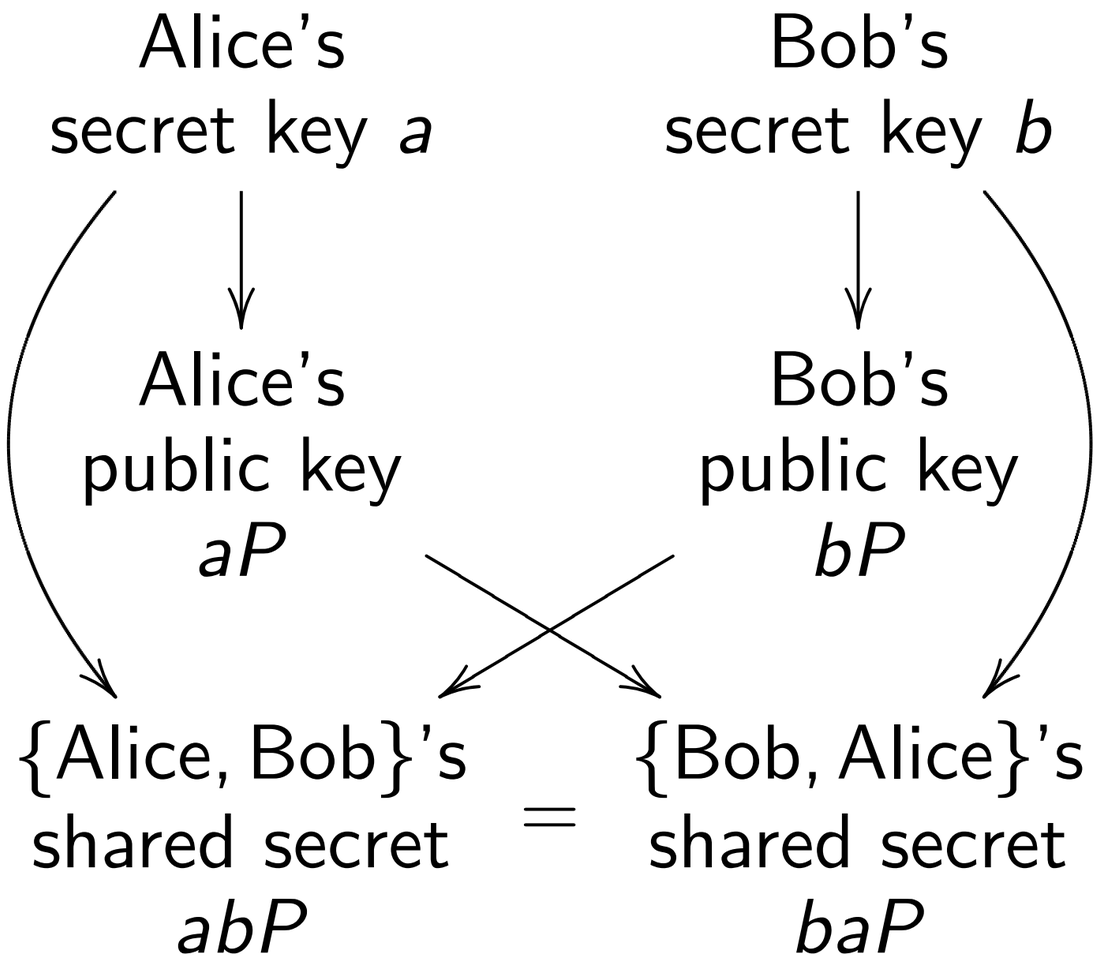

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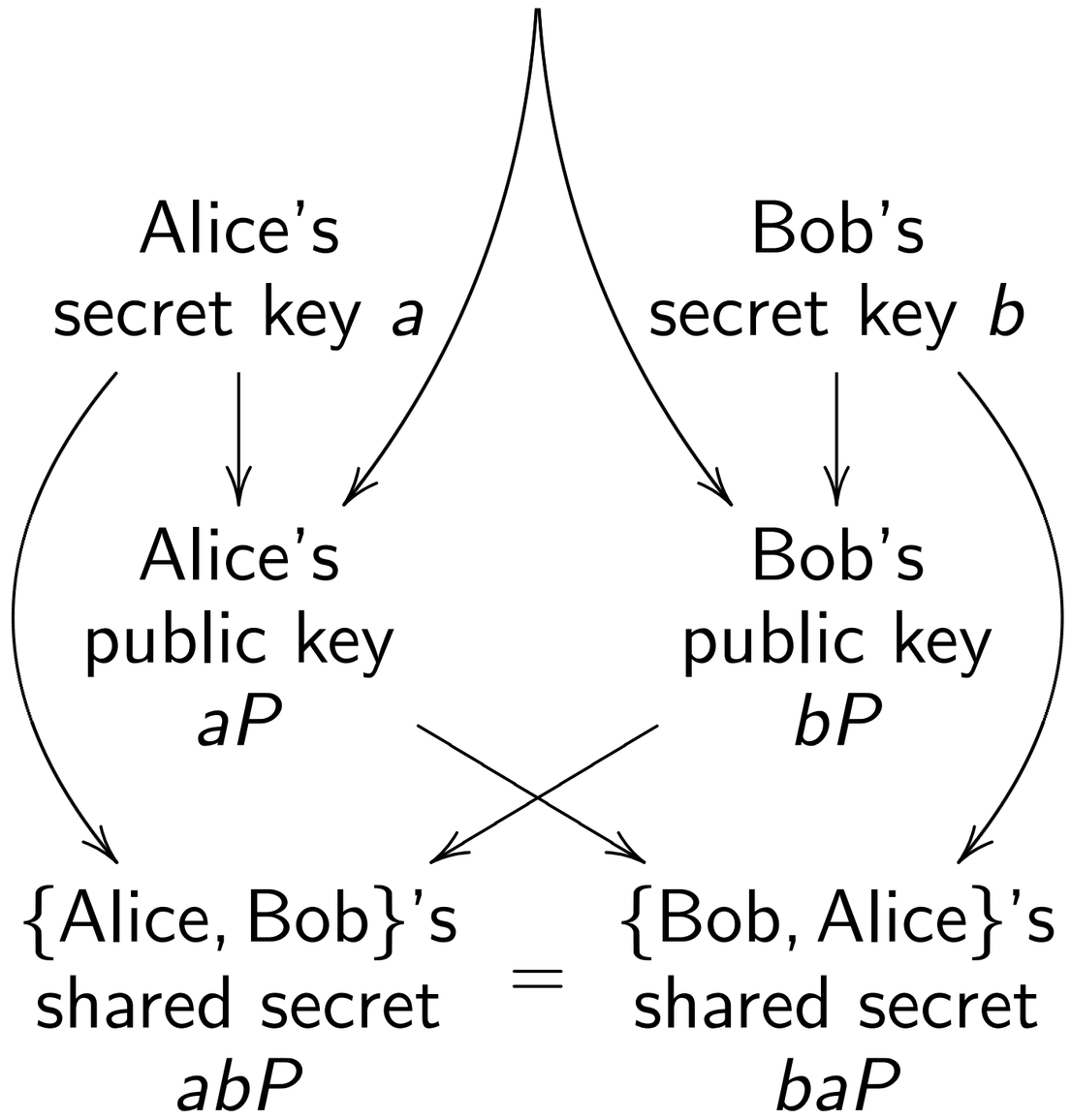

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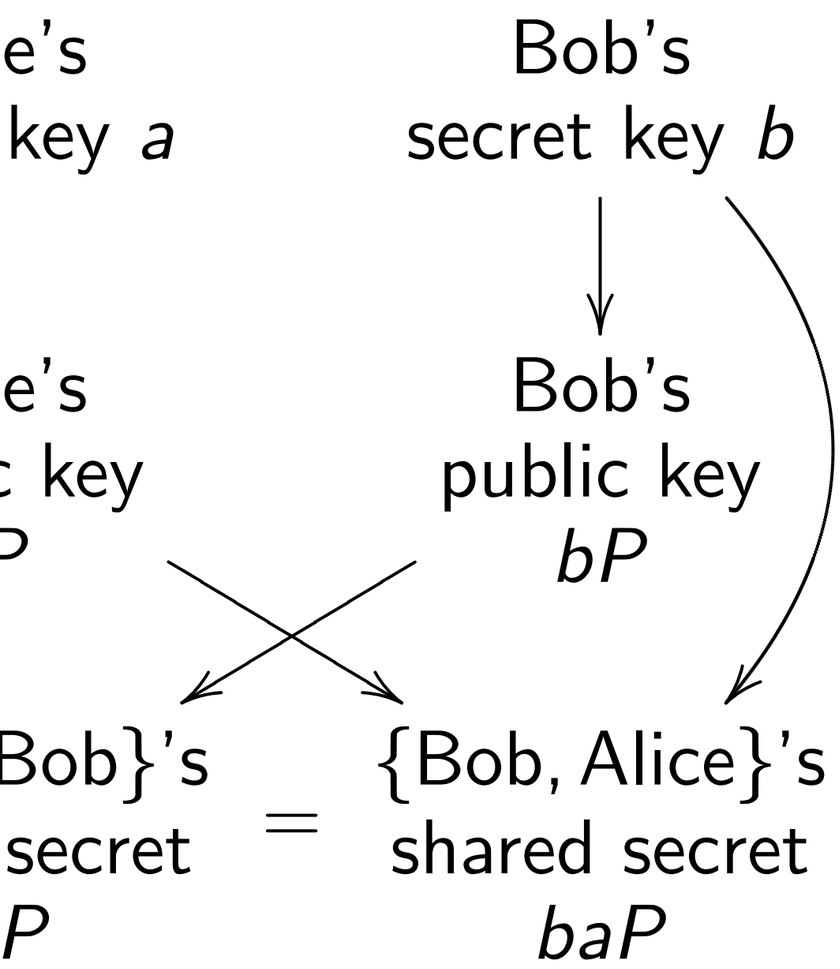

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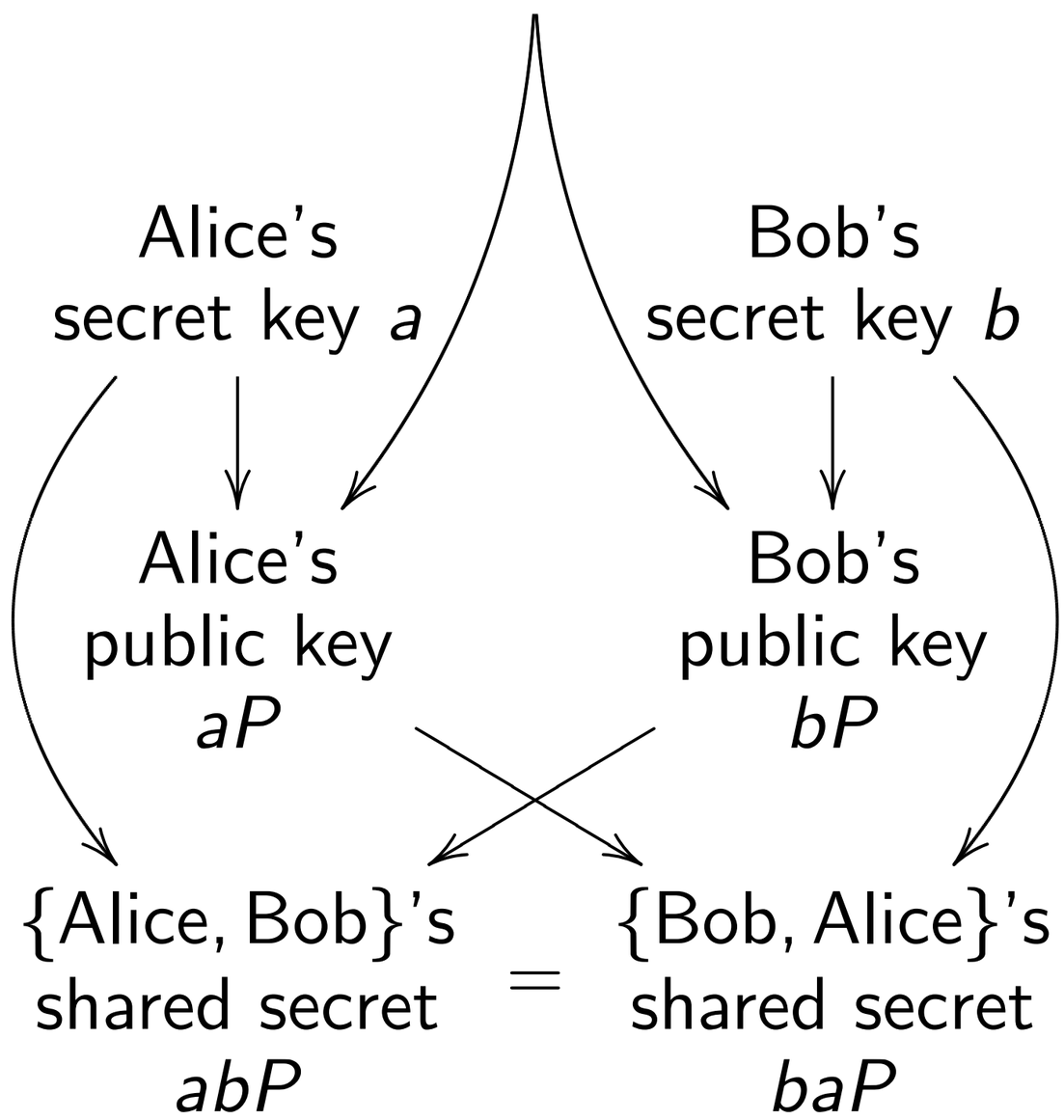

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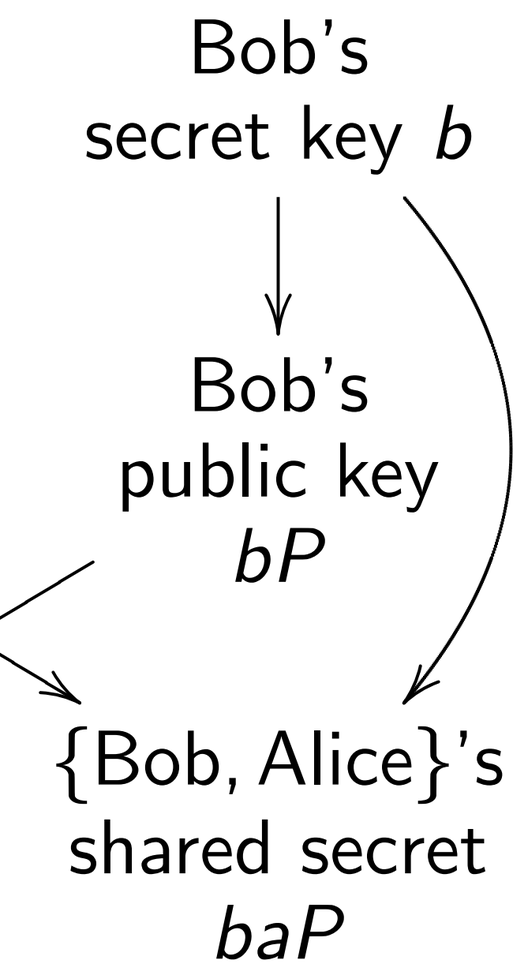

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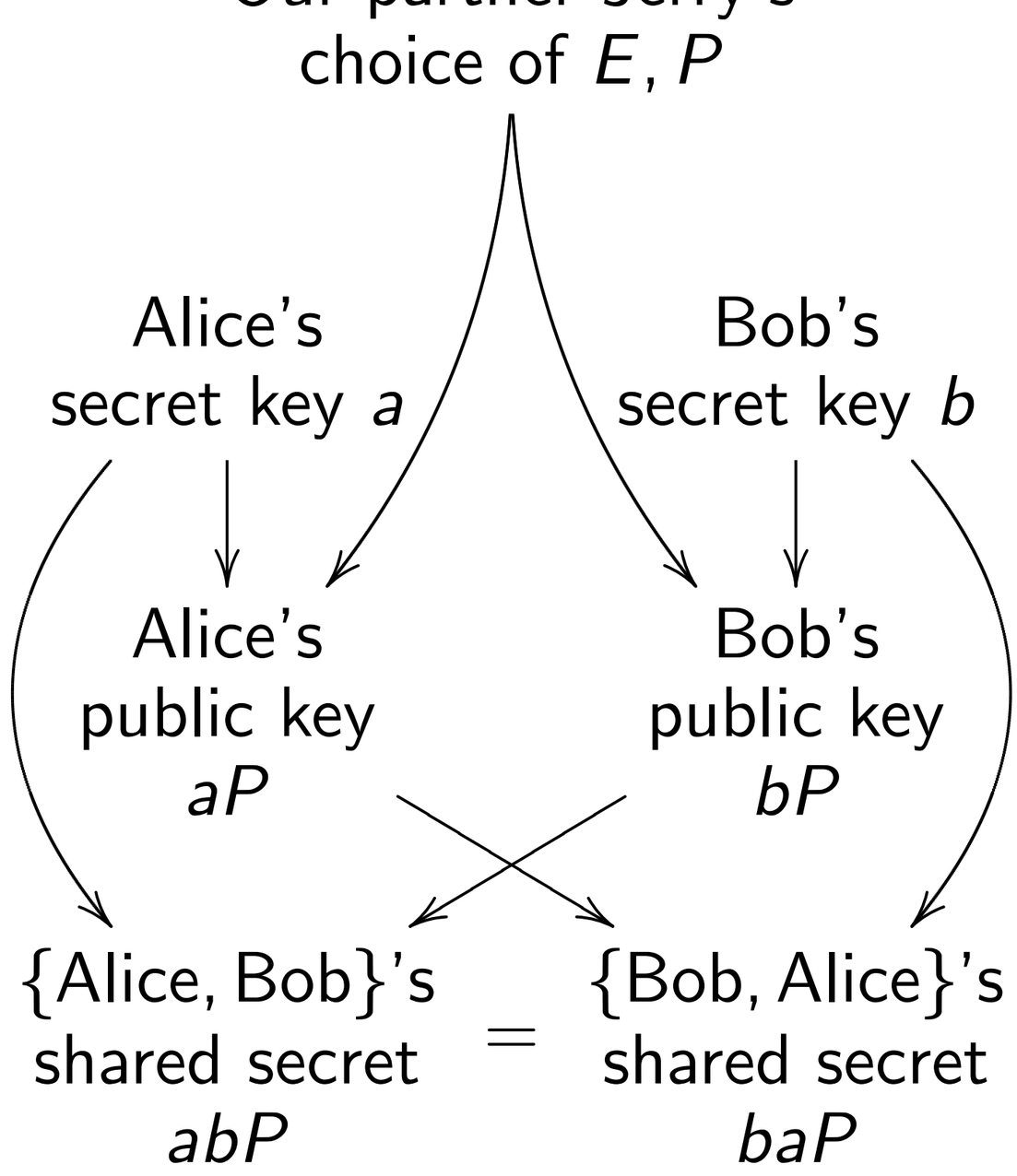

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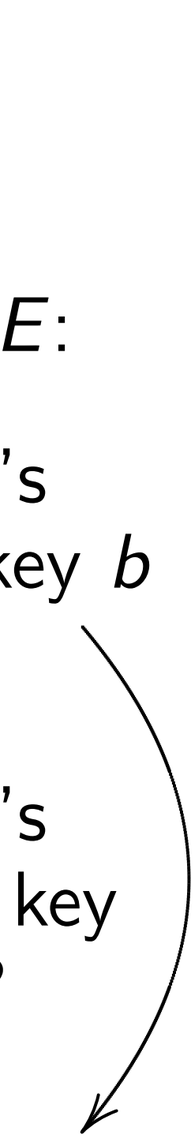

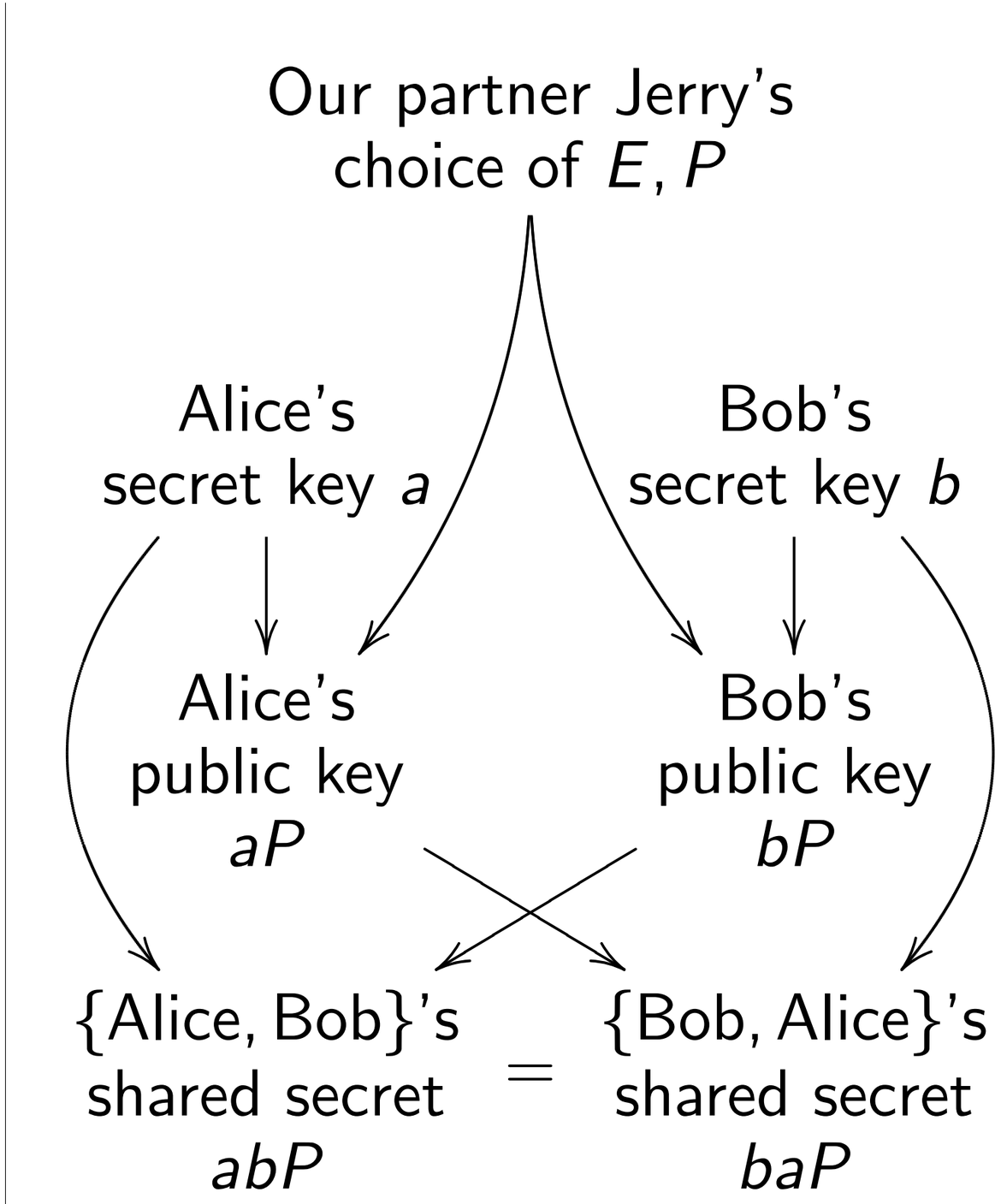

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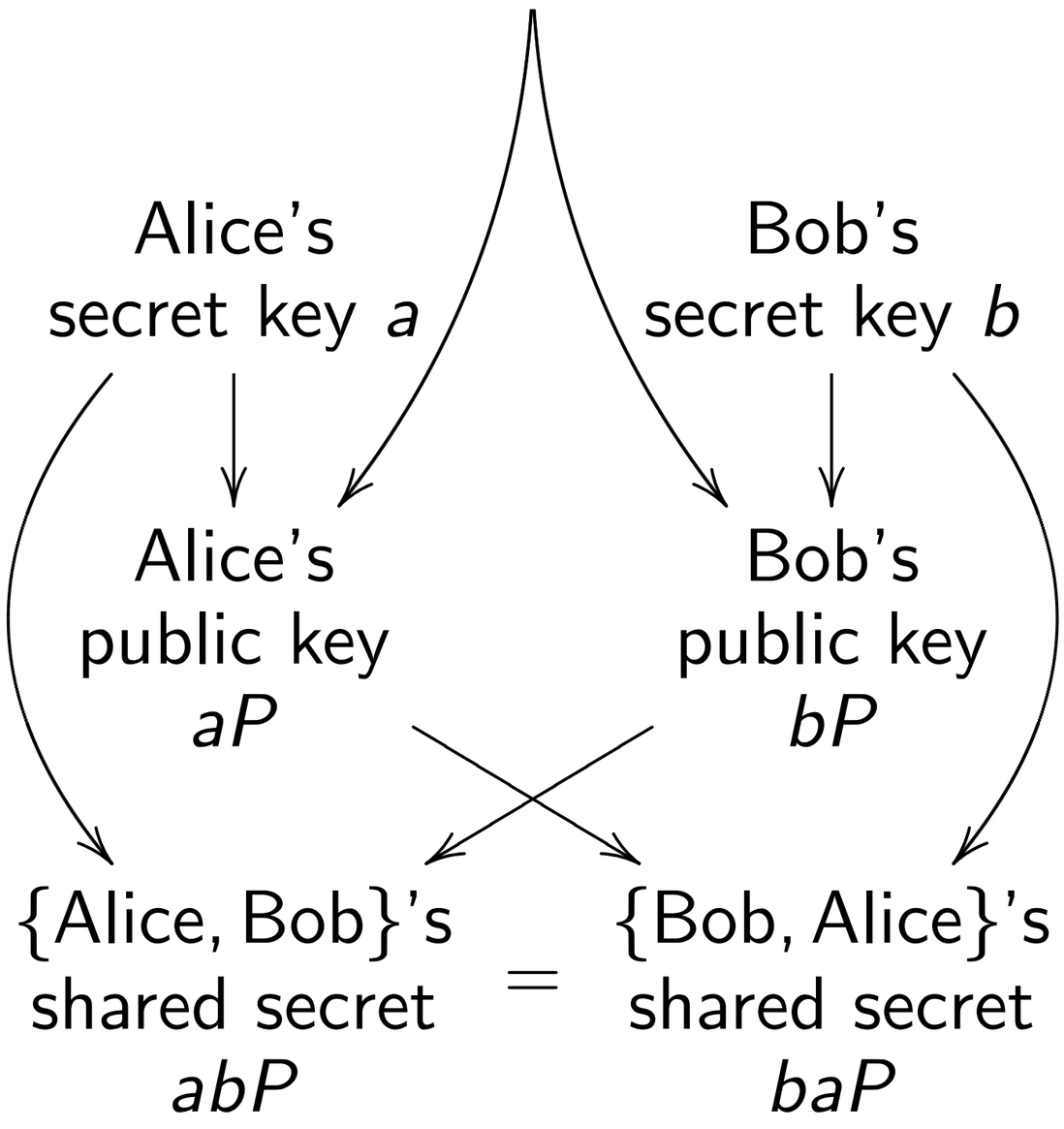

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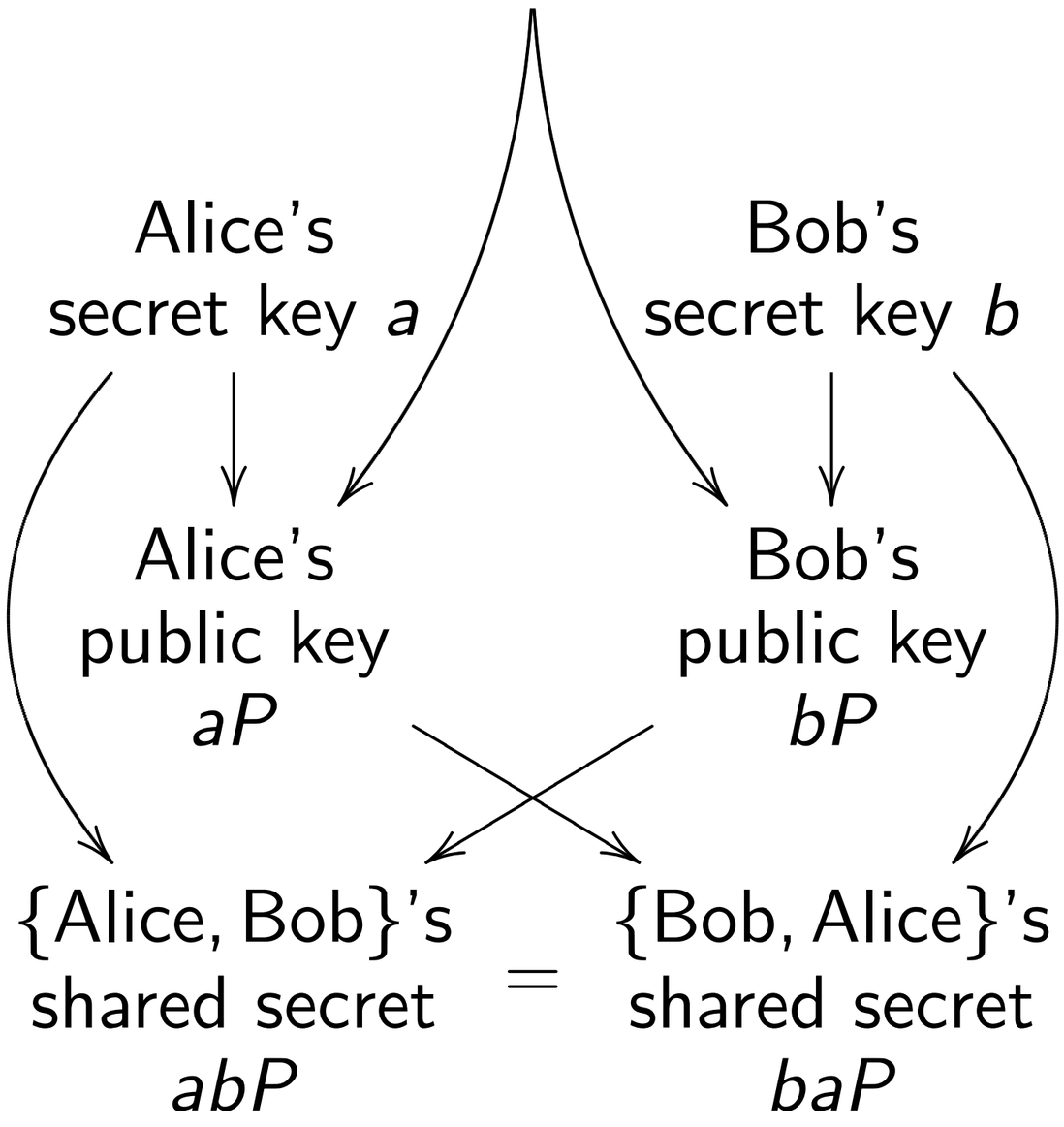

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Extensive ECC literature:
Pollard rho breaks small E,
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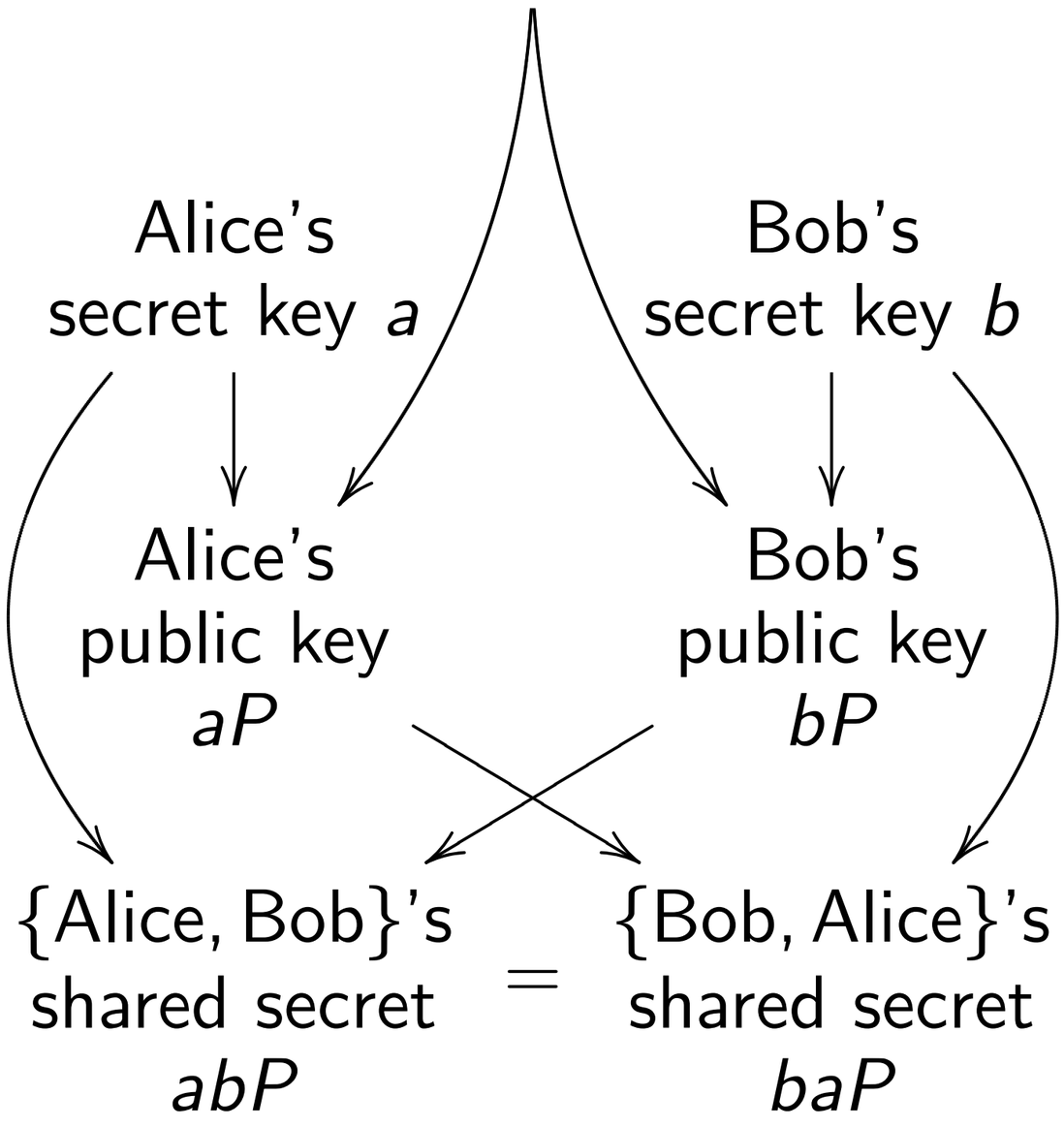
SmartASS breaks some E, etc.

Assume that public will accept any E not publicly broken.



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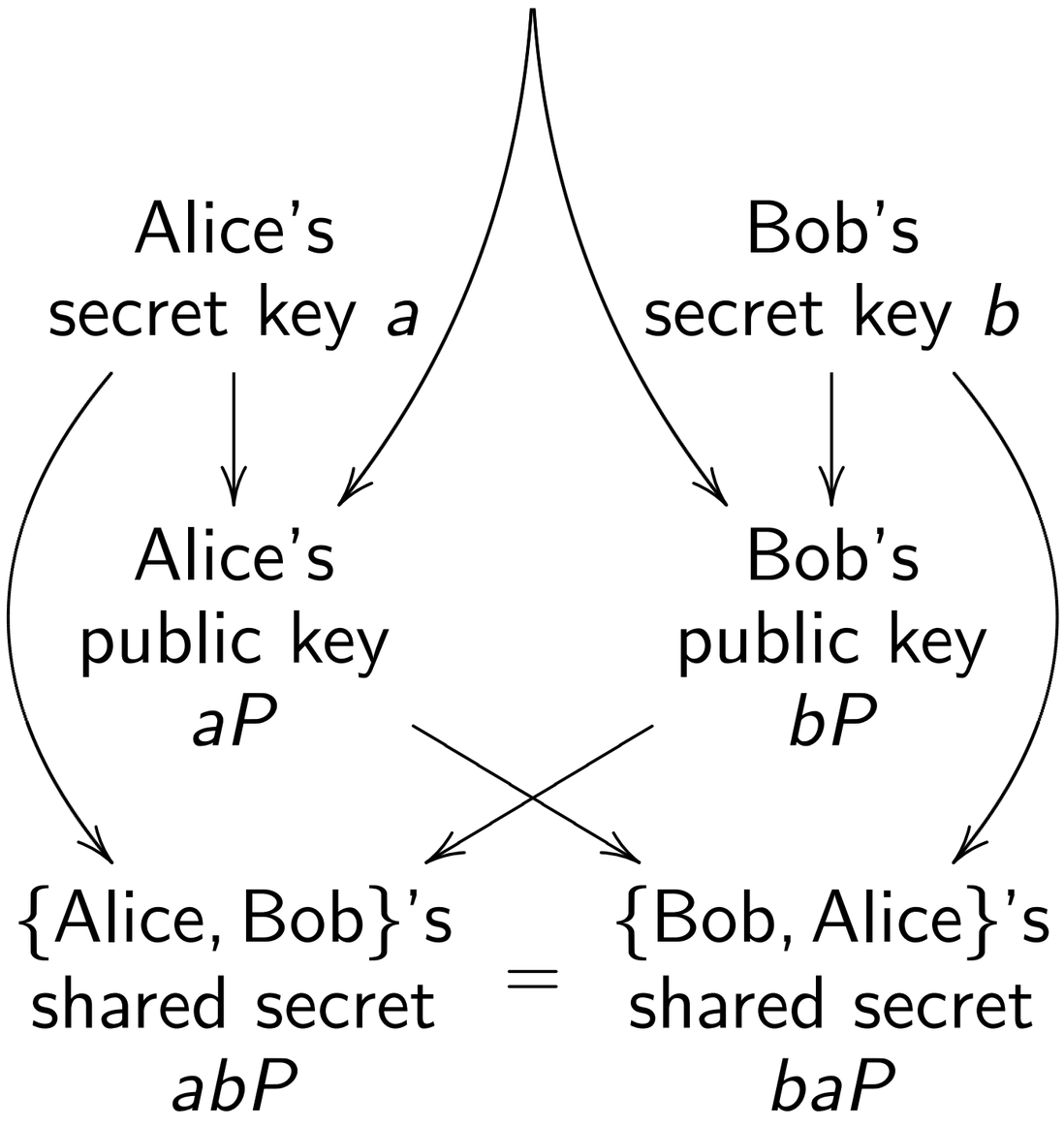

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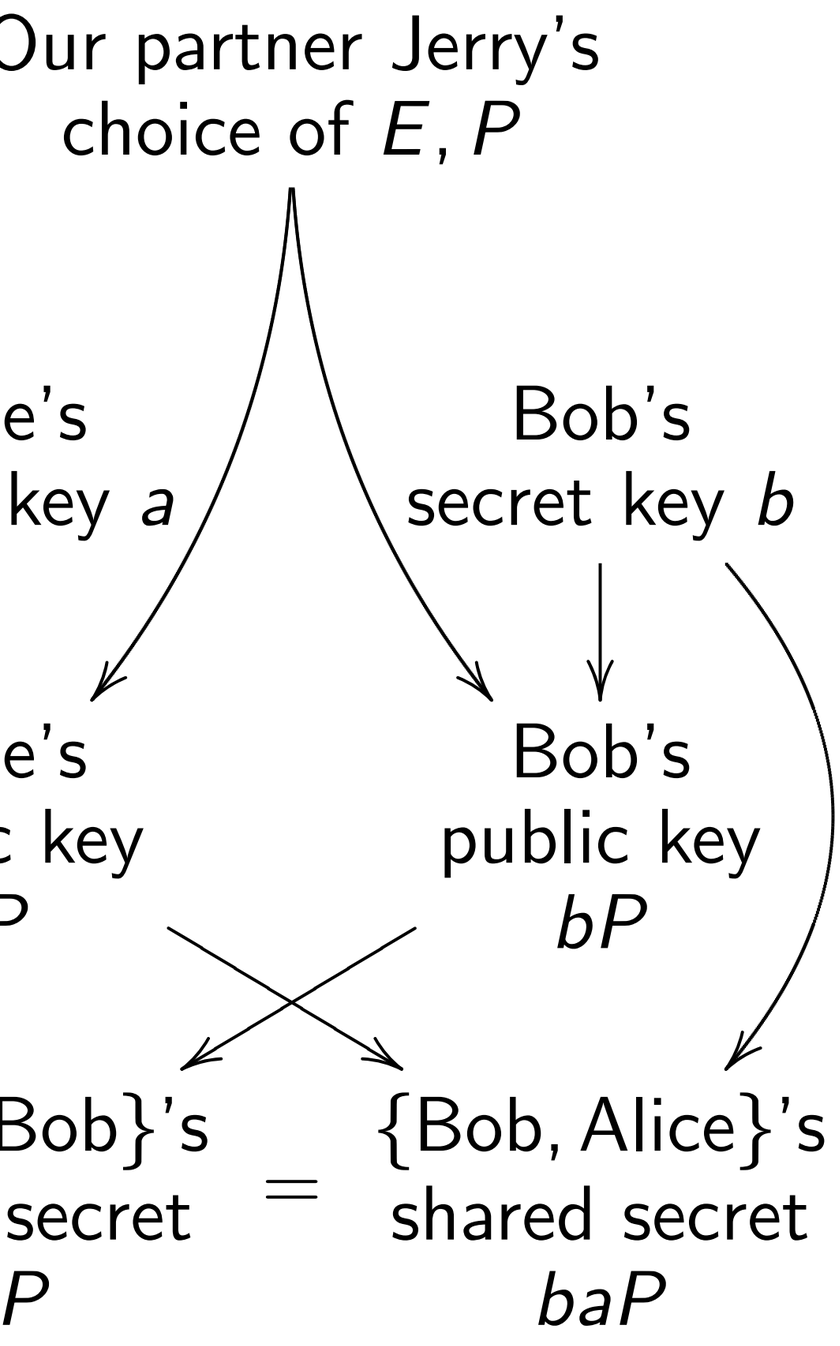

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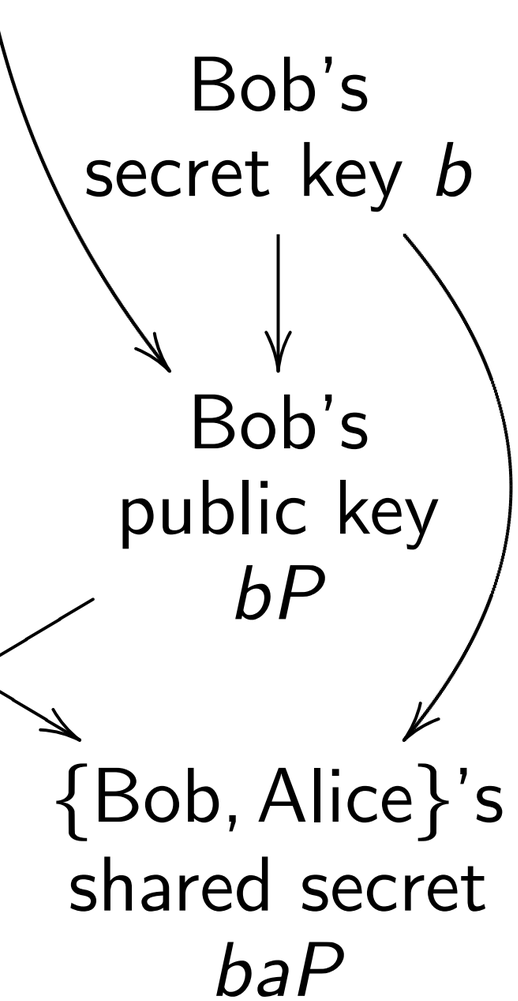

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The curve looks random;
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NIST defines curve E as $y^2 = x^3 - 3x + b$ where $b^2c = -27$; c is a hash of s; hash is SHA-1 concatenation.

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Optimized this computation on cluster of 41 GTX780 GPUs using H = Keccak. In 7 hours found "secure+twist-secure" $b =$ 0x

BADA55ECD8BBEAD3ADD6C534F92197DE

B47FCEB9BE7E0E702A8D1DD56B5D0B0C

mod NIST P-256.

Similarly found $b =$ 0x

BADA55ECFD9CA54C0738B8A6FB8CF4CC

F84E916D83D6DA1B78B622351E11AB4E

mod NIST P-224; and $b =$ 0x

BADA55EC3BE2AD1F9EEEA5881ECF95BB

F3AC392526F01D4CD13E684C63A17CC4

D5F271642AD8389913817A61006413D

mod NIST P-384.

efines curve E as
$-3x+b$ where
-27; c is a hash of s;
SHA-1 concatenation.

ott: "Consider now the
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mod NIST P-224; and $b =$ 0x
BADA55EC3BE2AD1F9EEEA5881ECF95BB
F3AC392526F01D4CD13E684C63A17CC4
D5F271642AD8389911381 7A61006413D
mod NIST P-384.

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BADA55EC3BE2AD1F9EEEA5881ECF95BB
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BADA55ECD8BBEAD3ADD6C534F92197DE
B47FCEB9BE7E0E702A8D1DD56B5D0B0C
mod NIST P-256.
Similarly found $b = \texttt{0x}$
BADA55ECFD9CA54C0738B8A6FB8CF4CC
F84E916D83D6DA1B78B622351E11AB4E
mod NIST P-224; and $b = \texttt{0x}$
BADA55EC3BE2AD1F9EEEA5881ECF95BB
F3AC392526F01D4CD13E684C63A17CC4
D5F271642AD8389911381 7A61006413D
mod NIST P-384.

Maybe in some countries
the public is more demandin

Optimized this computation on cluster of 41 GTX780 GPUs using H = Keccak. In 7 hours found "secure+twist-secure" $b = \texttt{0x}$

BADA55ECD8BBEAD3ADD6C534F92197DE

B47FCEB9BE7E0E702A8D1DD56B5D0B0C

mod NIST P-256.

Similarly found $b = \texttt{0x}$

BADA55ECFD9CA54C0738B8A6FB8CF4CC

F84E916D83D6DA1B78B622351E11AB4E

mod NIST P-224; and $b = \texttt{0x}$

BADA55EC3BE2AD1F9EEEA5881ECF95BB

F3AC392526F01D4CD13E684C63A17CC4

D5F271642AD8389911381 7A61006413D

mod NIST P-384.

Maybe in some countries the public is more demanding.

Optimized this computation on cluster of 41 GTX780 GPUs using H = Keccak. In 7 hours found "secure+twist-secure" $b =$ 0x BADA55ECD8BBEAD3ADD6C534F92197DE B47FCEB9BE7E0E702A8D1DD56B5D0B0C mod NIST P-256.
Similarly found $b =$ 0x BADA55ECFD9CA54C0738B8A6FB8CF4CC F84E916D83D6DA1B78B622351E11AB4E mod NIST P-224; and $b =$ 0x BADA55EC3BE2AD1F9EEEA5881ECF95BB F3AC392526F01D4CD13E684C63A17CC4 D5F271642AD8389913817A61006413D mod NIST P-384.

Maybe in some countries the public is more demanding.

Brainpool standard (2005): "The choice of the seeds from which the [NIST] curve parameters have been derived is not motivated leaving an essential part of the security analysis open. ... **Verifiably pseudo-random.** The [Brainpool] curves shall be generated in a pseudo-random manner using seeds that are generated in a systematic and comprehensive way."

ed this computation on
of 41 GTX780 GPUs using
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ST P-256.
y found $b = 0x$
CFD9CA54C0738B8A6FB8CF4CC
D83D6DA1B78B622351E11AB4E
ST P-224; and $b = 0x$
C3BE2AD1F9EEEA5881ECF95BB
526F01D4CD13E684C63A17CC4
42AD83899113817A61006413D
ST P-384.

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generated in a pseudo-random
manner using seeds that are
generated in a systematic and
comprehensive way."

```
import hashlib
def hash(seed): h
seedbytes = 20

p = 0xD7C134AA264
k = GF(p); R.<x> =

def secure(A,B):
  if k(B).is_squa
  n = EllipticCur
  return (n < p a
    and Integers(

def int2str(seed,
  return ''.join(

def str2int(seed)
  return Integer(

def update(seed):
  return int2str(

def fullhash(seed
  return str2int(

def real2str(seed
  return int2str(

nums = real2str(e
S = nums[2*seedby
while True:
  A = fullhash(S)
  if not (k(A)*x^
  S = update(S)
  B = fullhash(S)
  if not secure(A
  print 'p',hex(p
  print 'A',hex(A
  print 'B',hex(B
  break
```

mputation on
780 GPUs using
7 hours found
ure" $b =$ 0x
ADD6C534F92197DE
2A8D1DD56B5D0B0C

$=$ 0x
0738B8A6FB8CF4CC
78B622351E11AB4E
and $b =$ 0x
9EEEA5881ECF95BB
D13E684C63A17CC4
113817A61006413D

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. . . **Verifiably pseudo-random.**
The [Brainpool] curves shall be
generated in a pseudo-random
manner using seeds that are
generated in a systematic and
comprehensive way."

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardi
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%25

def str2int(seed):
  return Integer(seed.encode('hex'),16

def update(seed):
  return int2str(str2int(seed) + 1,len

def fullhash(seed):
  return str2int(hash(seed) + hash(upd

def real2str(seed,bytes):
  return int2str(Integer(floor(RealFie

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = upd
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); c
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

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manner using seeds that are
generated in a systematic and
comprehensive way.”

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*2

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

Maybe in some countries
the public is more demanding.

Brainpool standard (2005):
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The [Brainpool] curves shall be
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manner using seeds that are
generated in a systematic and
comprehensive way.”

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

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def secure(A,B):
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  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

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Output

```
p D7C134AA2643
A 2B98B906DC24
B 68AEC4BFE84C
```

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d (2005):
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IST] curve
been derived is
ving an essential
y analysis open.
eudo-random.
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```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

We carefully imple
the curve-generati
from the Brainpoo
Previous slide: 224

Output of this pro

```
p  D7C134AA264366862A18302575D1D7
A  2B98B906DC245F2916C03A2F953EA9
B  68AEC4BFE84C659EBB8B81DC39355A
```

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```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

We carefully implemented
the curve-generation proced
from the Brainpool standard
Previous slide: 224-bit proce

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4B
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA
```

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

We carefully implemented
the curve-generation procedure
from the Brainpool standard.
Previous slide: 224-bit procedure.

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

We carefully implemented the curve-generation procedure from the Brainpool standard.
Previous slide: 224-bit procedure.

Output of this procedure:

p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF

A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E

B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D

The standard 224-bit Brainpool curve **is not the same curve**:

p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF

A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43

B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

We carefully implemented the curve-generation procedure from the Brainpool standard. Previous slide: 224-bit procedure.

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure that **does** generate the standard Brainpool curve.

```
 = hashlib.sha1(); h.update(seed); return h.digest()


366862A18302575D1D787B09F075797DA89F57EC8C0FF
= k[]


re(): return False
ve([k(A),k(B)]).cardinality()
nd n.is_prime()
n)(p).multiplicative_order() * 100 >= n-1)

bytes):
[chr((seed//256^i)%256) for i in reversed(range(bytes))])

:
seed.encode('hex'),16)


str2int(seed) + 1,len(seed))

):
hash(seed) + hash(update(seed))) % 2^223

,bytes):
Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

xp(1)/16,7*seedbytes)
tes:3*seedbytes]


4+3).roots(): S = update(S); continue


,B): S = update(S); continue
).upper()
).upper()
).upper()
```

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the curve-generation procedure
from the Brainpool standard.
Previous slide: 224-bit procedure.

Output of this procedure:

p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF

A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E

B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D

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A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43

B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B

Next slide: a procedure
that **does** generate
the standard Brainpool curve.

```
import hashlib
def hash(seed): h
seedbytes = 20

p = 0xD7C134AA264
k = GF(p); R.<x> =

def secure(A,B):
  n = EllipticCur
  return (n < p a
    and Integers(

def int2str(seed,
  return ''.join(

def str2int(seed)
  return Integer(

def update(seed):
  return int2str(

def fullhash(seed
  return str2int(

def real2str(seed
  return int2str(

nums = real2str(e
S = nums[2*seedby
while True:
  A = fullhash(S)
  if not (k(A)*x^
  while True:
    S = update(S)
    B = fullhash(
    if not k(B).i
  if not secure(A
  print 'p',hex(p
  print 'A',hex(A
  print 'B',hex(B
  break
```

```
update(seed); return h.digest()

B09F075797DA89F57EC8C0FF



nality()

order() * 100 >= n-1)

6) for i in reversed(range(bytes))])

)

(seed))

ate(seed))) % 2^223

ld(8*bytes+8)(seed)*256^bytes)),bytes)



ate(S); continue

ontinue
```

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the curve-generation procedure
from the Brainpool standard.
Previous slide: 224-bit procedure.

Output of this procedure:

p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
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A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B

Next slide: a procedure
that **does** generate
the standard Brainpool curve.

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardi
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%25

def str2int(seed):
  return Integer(seed.encode('hex'),16

def update(seed):
  return int2str(str2int(seed) + 1,len

def fullhash(seed):
  return str2int(hash(seed) + hash(upd

def real2str(seed,bytes):
  return int2str(Integer(floor(RealFie

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = upd
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); c
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

```
h.digest()

C0FF

range(bytes))])

256^bytes)),bytes)
```

We carefully implemented
the curve-generation procedure
from the Brainpool standard.
Previous slide: 224-bit procedure.

Output of this procedure:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B  68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
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p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
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B  2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure
that **does** generate
the standard Brainpool curve.

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*2

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

We carefully implemented
the curve-generation procedure
from the Brainpool standard.
Previous slide: 224-bit procedure.

Output of this procedure:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B  68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool
curve **is not the same curve**:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B  2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure
that **does** generate
the standard Brainpool curve.

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
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def secure(A,B):
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    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
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def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

fully implemented
e-generation procedure
e Brainpool standard.
s slide: 224-bit procedure.

of this procedure:

66862A18302575D1D787B09F075797DA89F57EC8C0FF
5F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D

ndard 224-bit Brainpool
not the same curve:

66862A18302575D1D787B09F075797DA89F57EC8C0FF
6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
4138870713B1A92369E33E2135D266DBB372386C400B

de: a procedure
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dard Brainpool curve.

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

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cedure:

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9AE565C3253E8AEC4BFE84C659E
A2EBFA3870D98976FA2F17D2D8D

-bit Brainpool
same curve:

787B09F075797DA89F57EC8C0FF
3514E182AD8B0042A59CAD29F43
9E33E2135D266DBB372386C400B

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npool curve.

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

Did Brainpool che
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def int2str(seed,bytes):
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def str2int(seed):
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nums = real2str(exp(1)/16,7*seedbytes)
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  print 'p',hex(p).upper()
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  break
```

Did Brainpool check before
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Did they know before 2015?

Brainpool procedure is
advertised as “systematic”,
“comprehensive”, “complete
transparent”, etc. Surely we
say the same for *both* proce

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def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
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```

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    B = fullhash(S)
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  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
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  break
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Interesting Brainpool quote: "It is envisioned to provide additional curves on a regular basis."

```
 = hashlib.sha1(); h.update(seed); return h.digest()


366862A18302575D1D787B09F075797DA89F57EC8C0FF
= k[]


ve([k(A),k(B)]).cardinality()
nd n.is_prime()
n)(p).multiplicative_order() * 100 >= n-1)

bytes):
[chr((seed//256^i)%256) for i in reversed(range(bytes))])

:
seed.encode('hex'),16)


str2int(seed) + 1,len(seed))

):
hash(seed) + hash(update(seed))) % 2^223

,bytes):
Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

xp(1)/16,7*seedbytes)
tes:3*seedbytes]


4+3).roots(): S = update(S); continue


S)
s_square(): break
,B): S = update(S); continue
).upper()
).upper()
).upper()
```

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```
update(seed); return h.digest()

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6) for i in reversed(range(bytes))])

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(seed))

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ld(8*bytes+8)(seed)*256^bytes)),bytes)


ate(S); continue


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```

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We made a new 2
using standard NI

To avoid Brainpoo
complications of c
hash outputs: We
from SHA-1 to sta
maximum-security
Also upgraded to
maximum twist se

Brainpool uses exp
and $\arctan(1) = \pi$
uses $\sin(1)$, so we
We also used muc
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```
h.digest()

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range(bytes))])

256^bytes)),bytes)
```

Did Brainpool check before publication? After publication? Did they know before 2015?

Brainpool procedure is advertised as "systematic", "comprehensive", "completely transparent", etc. Surely we can say the same for *both* procedures.

Can quietly manipulate choice to take the weaker procedure.

Interesting Brainpool quote: "It is envisioned to provide additional curves on a regular basis."

We made a new 224-bit cur using standard NIST P-224

To avoid Brainpool's complications of concatenat hash outputs: We upgraded from SHA-1 to state-of-the- maximum-security SHA3-51 Also upgraded to requiring maximum twist security.

Brainpool uses $\exp(1) = e$ and $\arctan(1) = \pi/4$, and M uses $\sin(1)$, so we used $\cos($ We also used much simpler pattern of searching for seed

Did Brainpool check before publication? After publication? Did they know before 2015?

Brainpool procedure is advertised as "systematic", "comprehensive", "completely transparent", etc. Surely we can say the same for *both* procedures.

Can quietly manipulate choice to take the weaker procedure.

Interesting Brainpool quote: "It is envisioned to provide additional curves on a regular basis."

We made a new 224-bit curve using standard NIST P-224 prime.

To avoid Brainpool's complications of concatenating hash outputs: We upgraded from SHA-1 to state-of-the-art maximum-security SHA3-512. Also upgraded to requiring maximum twist security.

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```
import simplesha3
hash = simplesha3

p = 2^224 - 2^96 -
k = GF(p)
seedbytes = 20

def secure(A,B):
  n = EllipticCur
  return (n.is_pr
    and Integers(
    and Integers(

def int2str(seed,
  return ''.join(

def str2int(seed)
  return Integer(

def complement(se
  return ''.join(

def real2str(seed
  return int2str(

sizeofint = 4
nums = real2str(c
for counter in xr
  S = int2str(cou
  T = complement(
  A = str2int(has
  B = str2int(has
  if secure(A,B):
    print 'p',hex
    print 'A',hex
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    break
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```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardi
  return (n.is_prime() and (2*p+2-n).i
    and Integers(n)(p).multiplicative_
    and Integers(2*p+2-n)(p).multiplic

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%25

def str2int(seed):
  return Integer(seed.encode('hex'),16

def complement(seed):
  return ''.join([chr(255-ord(s)) for

def real2str(seed,bytes):
  return int2str(Integer(RealField(8*b

sizeofint = 4
nums = real2str(cos(1),seedbytes - siz
for counter in xrange(0,256^sizeofint)
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  T = complement(S)
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  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```

Output:

24-bit curve
ST P-224 prime.

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def real2str(seed,bytes):
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sizeofint = 4
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for counter in xrange(0,256^sizeofint):
  S = int2str(counter,sizeofint) + nums
  T = complement(S)
  A = str2int(hash(S))
  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```

Output: `7144BA12CE8A`

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```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

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  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
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    print 'B',hex(B).upper()
    break
```

Output: 7144BA12CE8A0C3BEFA053EDB

```
import simplesha3
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p = 2^224 - 2^96 + 1
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def secure(A,B):
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```

Output: `7144BA12CE8A0C3BEFA053EDBADA55...`

```
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```

Output: `7144BA12CE8A0C3BEFA053EDBADA55...`

We actually generated >1000000 curves for this prime, each having a Brainpool-like explanation, even without complicating hashing, seed search, etc.; e.g., `BADA55-VPR2-224` uses exp(1).

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See `bada55.cr.yp.to` for much more: full paper; scripts; detailed Brainpool analysis; manipulating "minimal" primes and curves (Microsoft "NUMS"); manipulating security criteria.