PQCHacks:
a gentle introduction to post-quantum cryptography

Daniel J. Bernstein[1,2] Tanja Lange[1]

[1]Technische Universiteit Eindhoven

[2]University of Illinois at Chicago

27 December 2015

Algorithms for Quantum Computation: Discrete Logarithms and Factoring

Peter W. Shor
AT&T Bell Labs
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA

Abstract

A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor. It is not clear whether this is still true when quantum mechanics is taken into consideration. Several researchers, starting with David Deutsch, have developed models for quantum mechanical computers and have investigated their compu-

[1, 2]. Although he did not ask whether quantum mechanics conferred extra power to computation, he did show that a Turing machine could be simulated by the reversible unitary evolution of a quantum process, which is a necessary prerequisite for quantum computation. Deutsch [9, 10] was the first to give an explicit model of quantum computation. He defined both quantum Turing machines and quantum circuits and investigated some of their properties.

The next part of this paper discusses how quantum com-


D-Wave
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- Can't store stable qubits.
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- Can't run Shor's algorithm.
- Can't run other quantum algorithms we care about.
- Hasn't managed to find any computation justifying its price.
- Hasn't managed to find any computation justifying 1% of its price.

. . . but universal quantum computers are coming, and are scary

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- Mark Ketchen, IBM Research, 2012, on quantum computing: "Were actually doing things that are making us think like, 'hey this isn't 50 years off, this is maybe just 10 years off, or 15 years off.' It's within reach."
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- Shor's algorithm solves in polynomial time:
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  - The discrete-logarithm problem in finite fields. DSA is dead.
  - The discrete-logarithm problem on elliptic curves. ECDSA is dead.
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- This breaks all current public-key cryptography on the Internet!
- Also, Grover's algorithm speeds up brute-force searches.
- Example: Only 2^{64} quantum operations to break AES-128;
  2^{128} quantum operations to break AES-256.

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- Very limited functionality: e.g., no public-key signatures.

Is there any hope? Yes!

Post-quantum crypto is crypto that resists attacks by quantum computers.

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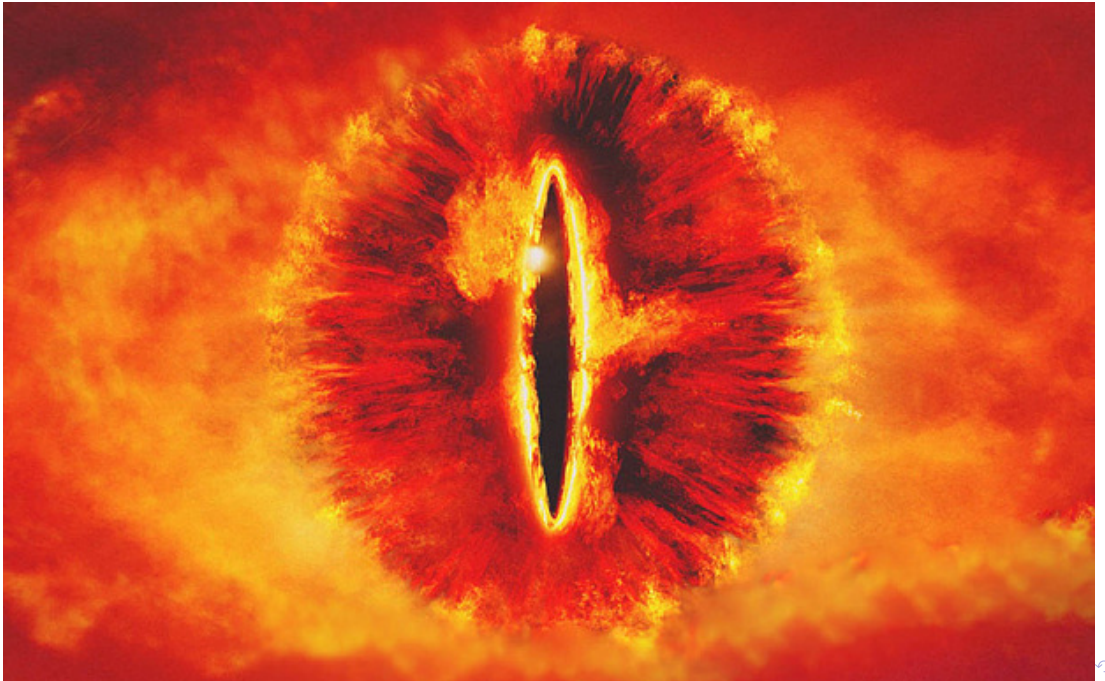
- New EU project, 2015–2018: PQCRYPTO, Post-Quantum Cryptography for Long-term Security.

PQCRYPTO
ICT-645622



NSA announcements

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August 19, 2015

> *IAD will initiate a transition to quantum resistant algorithms in the not too distant future.*

NSA comes late to the party and botches its grand entrance.

Confidence-inspiring crypto takes time to build

- Many stages of research from cryptographic design to deployment:
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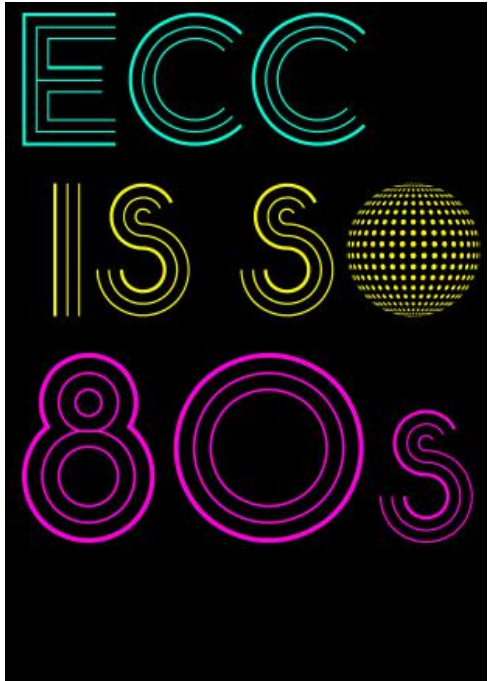
  - Integrate securely into real-world applications.
- Example: ECC introduced **1985**; big advantages over RSA.
  Robust ECC is starting to take over the Internet in **2015**.
- Post-quantum research can't wait for quantum computers!


ECC
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80s

Even higher urgency for long-term confidentiality

- Today's encrypted communication is being stored by attackers and will be decrypted years later with quantum computers. Danger for human-rights workers, medical records, journalists, security research, legal proceedings, state secrets, ...

Next slide:
Initial recommendations
of long-term secure post-quantum systems

Daniel Augot, Lejla Batina, Daniel J. Bernstein, Joppe Bos,
Johannes Buchmann, Wouter Castryck, Orr Dunkelman,
Tim Güneysu, Shay Gueron, Andreas Hülsing,
Tanja Lange, Mohamed Saied Emam Mohamed,
Christian Rechberger, Peter Schwabe, Nicolas Sendrier,
Frederik Vercauteren, Bo-Yin Yang

Initial recommendations

- **Symmetric encryption** Thoroughly analyzed, 256-bit keys:
  - AES-256
  - Salsa20 with a 256-bit key

  Evaluating: Serpent-256, ...
- **Symmetric authentication** Information-theoretic MACs:
  - GCM using a 96-bit nonce and a 128-bit authenticator
  - Poly1305
- **Public-key encryption** McEliece with binary Goppa codes:
  - length $n = 6960$, dimension $k = 5413$, $t = 119$ errors

  Evaluating: QC-MDPC, Stehlé-Steinfeld NTRU, ...
- **Public-key signatures** Hash-based (minimal assumptions):
  - XMSS with any of the parameters specified in CFRG draft
  - SPHINCS-256

  Evaluating: HFEv-, ...

Hash-based signatures

- Old idea: 1979 Lamport one-time signatures.
- Only one prerequisite: a good hash function.
- 1979 Merkle extends to more signatures.
- Many further improvements.
- Security thoroughly analyzed.

A signature scheme for empty messages: key generation

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```
from simplesha3 import sha3256

def keypair():
  secret = sha3256(os.urandom(32))
  public = sha3256(secret)
  return public,secret
```

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```

```
>>> import signempty
>>> pk,sk = signempty.keypair()
>>> binascii.hexlify(pk)
'a447bc8d7c661f85defcf1bbf8bad77bfc6191068a8b658c99c7ef4cbe37cf9f'
>>> binascii.hexlify(sk)
'a4a1334a6926d04c4aa7cd98231f4b644be90303e4090c358f2946f1c257687a'
```

A signature scheme for empty messages: signing, verification

```
def sign(message,secret):
  if message != '': raise Exception('nonempty message')
  signedmessage = secret
  return signedmessage

def open(signedmessage,public):
  if sha3256(signedmessage) != public: raise Exception('bad signature')
  message = ''
  return message
```

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def open(signedmessage,public):
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```

```
>>> sm = signempty.sign('',sk)
>>> signempty.open(sm,pk)
''
```

A signature scheme for 1-bit messages: key generation, signing

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```
import signempty

def keypair():
  p0,s0 = signempty.keypair()
  p1,s1 = signempty.keypair()
  return p0+p1,s0+s1

def sign(message,secret):
  if message == 0: return '0' + signempty.sign('',secret[0:32])
  if message == 1: return '1' + signempty.sign('',secret[32:64])
  raise Exception('message must be 0 or 1')
```

A signature scheme for 1-bit messages: verification

```
def open(signedmessage,public):
  if signedmessage[0] == '0':
    signempty.open(signedmessage[1:],public[0:32])
    return 0
  if signedmessage[0] == '1':
    signempty.open(signedmessage[1:],public[32:64])
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    return 1
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```

```
>>> import signbit
>>> pk,sk = signbit.keypair()
>>> sm = signbit.sign(1,sk)
>>> signbit.open(sm,pk)
1
```

A signature scheme for 4-bit messages: key generation

```
import signbit

def keypair():
  p0,s0 = signbit.keypair()
  p1,s1 = signbit.keypair()
  p2,s2 = signbit.keypair()
  p3,s3 = signbit.keypair()
  return p0+p1+p2+p3,s0+s1+s2+s3
```

A signature scheme for 4-bit messages: signing

```
def sign(m,secret):
  if type(m) != int: raise Exception('message must be int')
  if m < 0 or m > 15: raise Exception('message must be between 0 and 15')
  sm0 = signbit.sign(1 & (m >> 0),secret[0:64])
  sm1 = signbit.sign(1 & (m >> 1),secret[64:128])
  sm2 = signbit.sign(1 & (m >> 2),secret[128:192])
  sm3 = signbit.sign(1 & (m >> 3),secret[192:256])
  return sm0+sm1+sm2+sm3
```

A signature scheme for 4-bit messages: verification

```
def open(sm,public):
  m0 = signbit.open(sm[0:33],public[0:64])
  m1 = signbit.open(sm[33:66],public[64:128])
  m2 = signbit.open(sm[66:99],public[128:192])
  m3 = signbit.open(sm[99:132],public[192:256])
  return m0 + 2*m1 + 4*m2 + 8*m3
```

Achtung: Do not use one secret key to sign two messages!

```
>>> import sign4bits
>>> pk,sk = sign4bits.keypair()
>>> sm11 = sign4bits.sign(11,sk)
>>> sign4bits.open(sm11,pk)
11
>>> sm7 = sign4bits.sign(7,sk)
>>> sign4bits.open(sm7,pk)
7
>>> forgery = sm7[:99] + sm11[99:]
>>> sign4bits.open(forgery,pk)
15
```

Lamport's 1-time signature system

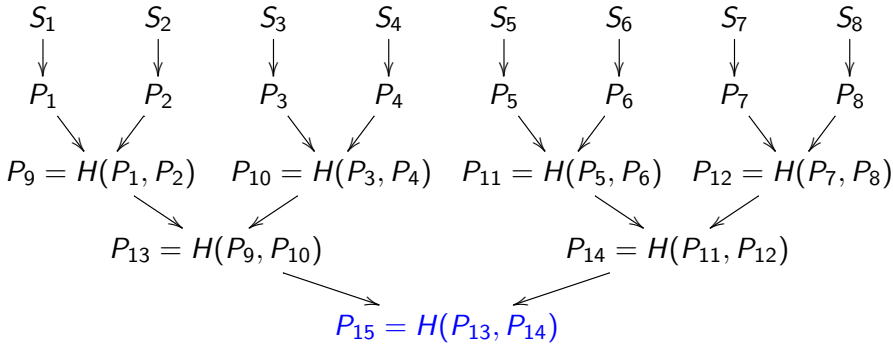
- Scale up to 256-bit messages.
- Sign arbitrary-length message by signing its 256-bit hash:

```
def sign(message,secret):
  h = sha3256(message)
  hbits = [1 & (ord(h[i/8])>>(i%8)) for i in range(256)]
  sigs = [signbit.sign(hbits[i],secret[64*i:64*i+64])
          for i in range(256)]
  return ''.join(sigs) + message
```

- Space improvement: "Winternitz signatures".

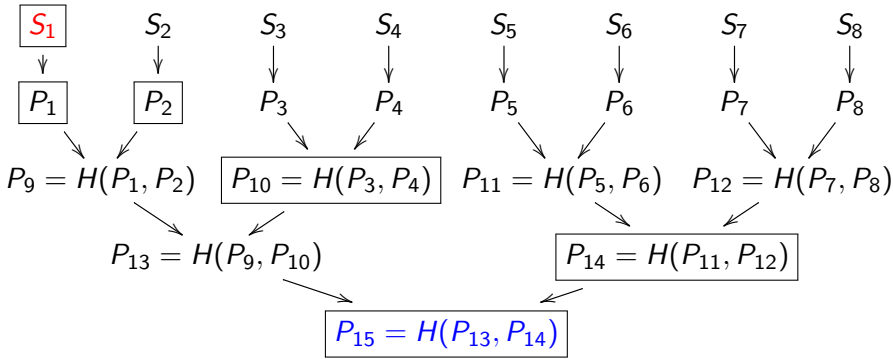
Merkle's (e.g.) 8-time signature system

Hash 8 Lamport one-time public keys into a single Merkle public key P_{15}.



Signature in 8-time Merkle hash tree

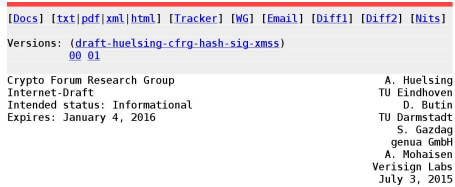
Signature of first message: $(\texttt{sign}(m, S_1), P_1, P_2, P_{10}, P_{14})$.



Pros and cons

Pros:

- Post-quantum
- Need secure hash function, nothing else
- Small public key
- Security well understood
- Fast
- Proposed for standards:
  https://tools.ietf.org/html/
  draft-irtf-cfrg-xmss-hash-based-signatures-01



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Versions: (draft-huelsing-cfrg-hash-sig-xmss)
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Crypto Forum Research Group
Internet-Draft
Intended status: Informational
Expires: January 4, 2016

A. Huelsing
TU Eindhoven
D. Butin
TU Darmstadt
S. Gazdag
genua GmbH
A. Mohaisen
Verisign Labs
July 3, 2015

XMSS: Extended Hash-Based Signatures
draft-irtf-cfrg-xmss-hash-based-signatures-01

Abstract

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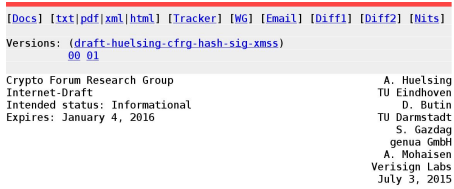

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Cons:

- Biggish signature
- Stateful. Adam Langley: “for most environments it’s a huge foot-cannon.”


ELIMINATE
THE STATE

Stateless hash-based signatures

- Idea from 1987 Goldreich:
  - Signer builds huge tree of certificate authorities.
  - Signature includes certificate chain.
  - Each CA is a hash of master secret and tree position.
    This is deterministic, so don't need to store results.
  - **Random** bottom-level CA signs message.
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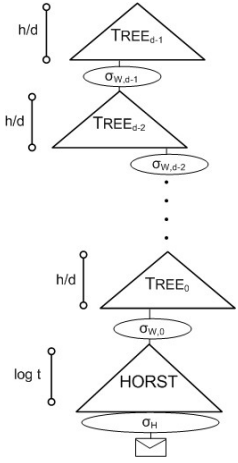

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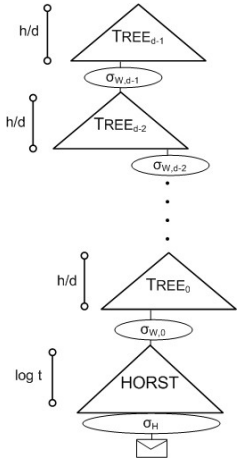

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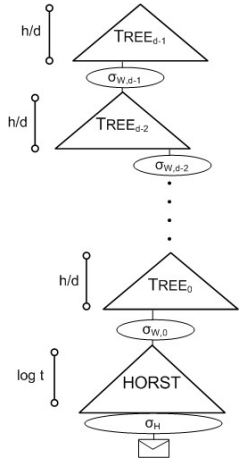
- 0.041 MB: SPHINCS signature, new optimization of Goldreich.
  Designed for 2^{128} post-quantum security. sphincs.cr.yp.to



Error correction

- Digital media is exposed to memory corruption.
- Many systems check whether data was corrupted in transit:
  - ISBN numbers have check digit to detect corruption.
  - ECC RAM detects up to two errors and can correct one error.
    64 bits are stored as 72 bits: extra 8 bits for checks and recovery.
- In general, k bits of data get stored in n bits, adding some redundancy.
- If no error occurred, these n bits satisfy $n - k$ parity check equations;
  else can correct errors from the error pattern.
- Good codes can correct many errors without blowing up storage too much;
  offer guarantee to correct t errors (often can correct or at least detect more).
- To represent these check equations we need a matrix.

Hamming code

Parity check matrix ($n = 7, k = 4$):

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

An error-free string of 7 bits $\mathbf{b} = (b_0, b_1, b_2, b_3, b_4, b_5, b_6)$ satisfies these three equations:

$$\begin{array}{rrrrrrrcl} b_0 & & & +b_3 & +b_4 & & +b_6 & = & 0 \\ & b_1 & & +b_3 & & +b_5 & +b_6 & = & 0 \\ & & b_2 & & +b_4 & +b_5 & +b_6 & = & 0 \end{array}$$

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In math notation, the failure pattern is $H \cdot \mathbf{b}$.

Coding theory

- Names: code word $\mathbf{c}$, error vector $\mathbf{e}$, received word $\mathbf{b} = \mathbf{c} + \mathbf{e}$.
- Very common to transform the matrix so that the left part has just 1 on the diagonal (no need to store that).

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

- Many special constructions discovered in 65 years of coding theory:
  - Large matrix H.
  - Fast decoding algorithm to find $\mathbf{e}$ given $\mathbf{s} = H \cdot (\mathbf{c} + \mathbf{e})$, whenever $\mathbf{e}$ doesn't have too many bits set.
- Given large H, usually very hard to find fast decoding algorithm.
- Use this difference in complexities for encryption.

Code-based encryption

- 1971 Goppa: Fast decoders for many matrices H.
- 1978 McEliece: Use Goppa codes for public-key cryptography.
  - Original parameters designed for 2^{64} security.
  - 2008 Bernstein–Lange–Peters: broken in $\approx 2^{60}$ cycles.
  - Easily scale up for higher security.
- 1986 Niederreiter: Simplified and smaller version of McEliece.
  - Public key: H with 1's on the diagonal.
  - Secret key: the fast Goppa decoder.
  - Encryption: Randomly generate e with t bits set.
    Send $H \cdot e$.
  - Use hash of e to encrypt message with symmetric crypto (with 256 bits key).
- Very fast constant-time decryption: `http://binary.cr.yp.to/mcbits.html`.

Security analysis

- Some papers studying algorithms for attackers:
  1962 Prange; 1981 Omura; 1988 Lee–Brickell; 1988 Leon; 1989 Krouk;
  1989 Stern; 1989 Dumer; 1990 Coffey–Goodman; 1990 van Tilburg; 1991 Dumer;
  1991 Coffey–Goodman–Farrell; 1993 Chabanne–Courteau; 1993 Chabaud;
  1994 van Tilburg; 1994 Canteaut–Chabanne;
  1998 Canteaut–Chabaud; 1998 Canteaut–Sendrier; 2008 Bernstein–Lange–Peters;
  2009 Bernstein–Lange–Peters–van Tilborg; 2009 Bernstein (post-quantum);
  2009 Finiasz–Sendrier; 2010 Bernstein–Lange–Peters; 2011 May–Meurer–Thomae;
  2011 Becker–Coron–Joux; 2012 Becker–Joux–May–Meurer;
  2013 Bernstein–Jeffery–Lange–Meurer (post-quantum); 2015 May–Ozerov.

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- 256 KB public key for 2^{146} pre-quantum security.
- 512 KB public key for 2^{187} pre-quantum security.
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- Post-quantum (Grover): below 2^{263}, above 2^{131}.

Many more post-quantum suggestions

- QC-MDPC: variant with much smaller keys, but is it secure?
- Many more code-based systems. Some broken, some not.
- NTRU: 1990s "lattice-based" system, similar to QC-MDPC.
  Security story less stable than code-based cryptography.
- Many more lattice-based systems. Some broken, some not.
  e.g., 2014 quantum break of 2009 Smart–Vercauteren system.
- Many multivariate-quadratic systems. Some broken, some not.
  Highlight: very small signatures.
- More exotic possibility that needs analysis: isogeny-based crypto.
  Highlight: supports DH.

Further resources

- https://pqcrypto.org: Our survey site.
  - Many pointers: e.g., PQCrypto 2016.
  - Bibliography for 4 major PQC systemss. Help us keep this up to date!
- https://pqcrypto.eu.org: PQCRYPTO EU project. Coming soon:
  - Expert recommendations.
  - Free software libraries.
  - More benchmarking to compare cryptosystems.
  - 2017: workshop and spring/summer school.
- https://twitter.com/pqc_eu: PQCRYPTO Twitter feed.
- **You!**
  - Get used to post-quantum cryptosystems.
  - Improve; implement; integrate into real-world systems.